Analysis of Selected Literature on Co-simulation

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Abstract. In this paper, a selection of literature on co-simulation has been classified along various aspects and analyzed to illuminate more and less popular topics. General divisions along the works' main topic, regarded model descriptions and considered coupling methods are presented, therein strong and loose coupling approaches, multirate schemes, iterative methods, or adaptive approaches and the progression of these shares over time, revealing tendencies towards more popular methods and areas that invite additional research.

Introduction

Co-simulation has become an important instrument in the area of modeling and simulation as the complexity of considered systems increases permanently. Depending on the disciplines they originate from as well as the level and depth of development, co-simulation methods can be classified by various different means, some of which have been presented in [1]. Consequently, literature on co-simulation can be associated with attributes of this structure and further aspects such as theoretical focus or the number of coupled subsystems.

The literature considered in this paper has been accumulated by strategic search in different catalogues and search machines (by the keywords “co-simulation", “cooperative simulation", “coupled simulation", “hybrid simulation", “multi-level simulation", “hierarchical (co-)simulation”), contributions to attended conferences, recommendations by fellow researchers in the area of co-simulation and citations in papers found in the first iteration and again, in these, etc.

The selected work has been classified in two iterations. First, every publication has been carefully read, summed up and assigned certain properties in evidence, such as used model description, coupling algorithm, and application. These properties have then been complemented by methods for structuring developed in the classification found in [1]. In the second iteration, each work has been studied for these properties (or indisputable indications of them) and classified accordingly. A complete list of these assignments is available in [2]. It is important to bear in mind that some properties could not be defined since the respective authors neither explicitly mention them nor could the property be found out from the paper's context.

1 Publications over the years

Selected by the method described above, a total of 139 publications remains to be considered in the statistical analysis. Regarding the publication dates of these, we observe that although multirate and cooperative simulation has been touched incidentally before the millennium (starting in 1960), research in this area has seen a significant upswing since. Table 1 lists the number of papers per five-year time frame, where publications before 2000 are grouped due to their small total number.

Table 1: Number of papers on co-simulation per time frame.

<2000	2000-2004	2005-2009	2010-2014	2015-2019
17	24	20	38	40

2 Main emphasis in the literature

Depending on the emphasis of the respective publication, each is assigned one topic out of “theory”, “application”, “survey”, “standard” or “framework”. This label describes whether the publication focuses *mainly* on theory, application, etc., not solely. Those where theory and application are quite balanced have been classified “both theory and application”. A majority (63%) of publications mainly covers theoretical aspects. 21 (15%) are applications of already known methods and for eight papers (6%), the theoretical and applied part are quite evenly matched. Eight publications are pure surveys (some limited to an application area of interest), two describe a standard (the HLA [3] and FMI [4]), and thirteen present a framework. These shares, however, have changed throughout the years, as Figure 1 illustrates.

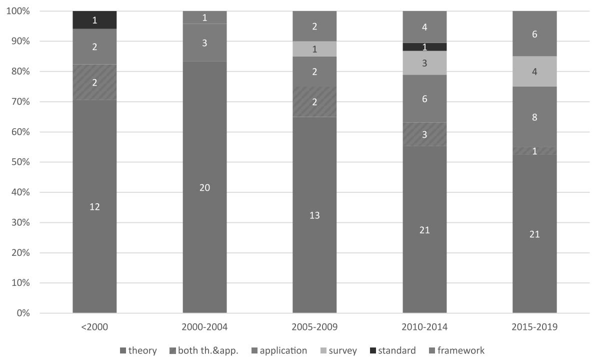

Figure 1: Variation of the main emphases' shares over time regarding five-year intervals. Numbers in the bars denote the quantity of publications per category in the respective time frame. Corresponding percentages can be read off the left-hand axis.

The outstandingly high share of theoretical papers in the years 2000-2004 amounts to more than 80%. The development of frameworks seems to have become more popular over the last two decades. Surveys start to occur in 2009 and have become more frequent ever since. This might simply be explained by the increasing amount of research in this area, thus raising the necessity of aggregating studies.

2.1 Theoretical subcharacterization of the literature

Although only 95 of the 139 publications exhibit a mainly theoretical or shared applied and theoretical focus, 122 (88%) consider at least one theoretical aspect. We further distinguish the following subcategories: error estimates, stability properties, coupling methods, performance, debugging, formalisms and classifications. Of all papers with theoretical aspect, a vast majority of 80% is investigating coupling methods, cf. Figure 2. Of these, over one third (36%) is dealing with the stability of these methods and 28% with error estimation (cf. Table 2). Note that these are not exclusive: again about one third thereof (12% of all that study coupling methods) considers both investigations on stability and error estimates for the regarded coupling methods. About one tenth is analyzing performance properties of these methods.

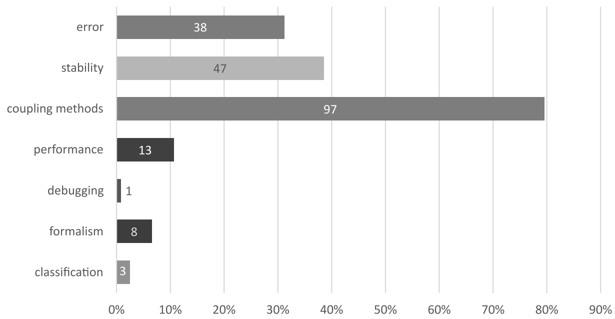

Figure 2: Share of publications considering specific theoretical subcategories. Absolute numbers are given in the bars, percentages are found on the horizontal axis.

Table 2: Number of publications on coupling methods combined with further theoretical categories (percentage of "coupling methods" in parentheses)

coupling methods and			
error	stability	performance	no further category
35 (36%)	27 (28%)	11 (11%)	40 (41%)

Taking a look at the overall parts of theoretical subcategories again in Figure 2, "stability" and "error" are the next most commonly investigated aspects with 39 and 31 percent, respectively. Within those, 19 publications consider both. 8 publications (11%) describe formalisms. These have mainly been published in the last five years. All three papers presenting classifications have also only come out in this time frame. These circumstances again relate to the increase of research and the variety of methods in co-simulation over the last two decades. In contrast, the share of publications including investigations on stability properties and error estimates, which had its peak in the time frame from 2005 to 2009 (at the expense of coupling methods), has dwindled in the last years.

3 Distinction by the state of development

Research on co-simulation methods originates from needs at different stages in the development of simulation models. On the one hand, facing a complex real system with partial systems differing to a great extent in their modeling requirements, these have to be approached with different techniques. The resulting separate simulations need to be coupled thereafter

to sufficiently represent the whole system (*integrate-and-collaborate*). On the other hand, complex systems within one physical domain may be described by one mathematical model that may consist of many equations which may be solved more efficiently by parallelization, possibly divided depending on stiffness. Again, the partial models will be simulated individually and coupled again in an overall simulation, but in this case, the need for separation arises from consideration of the system on the (mathematical) model level (*divde-and-conquer*). Whether a decomposition or collaborative coupling approach is used can be defined for 121 of the regarded publications. 60% of these are approaching their task with an integrate-and-collaborate strategy, 38% use divide-and-conquer methods and 2% utilize or compare both.
Over time, the share of works on integrate-and-collaborate approaches increases noticeably, from 25% to 80%. This consorts with the increase of surveys, standard and framework descriptions in the literature. These, as Figure 3 shows, exclusively represent an integrate-and-collaborate point of view.

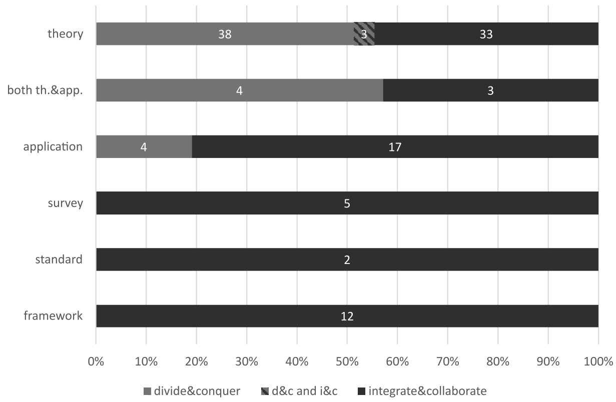

Figure 3: Decomposition of coupling point of view per main topic. The shares of divide-and-conquer and integrate-and-collaborative approaches are given separately for publications assigned to a specific main topic.

The illustration of this cross-connection further reveals that publications mainly presenting an application mostly address an integrate-and-collaborate problem, while for theoretical papers, shares of divide-and-conquer and integrate-and-collaborate approaches are quite balanced.

4 Distinction by field of application

The need for co-simulation arises in various fields of application which are closely related to the kind of coupling methods the systems are approached with and lead to varying developments. Note that many theory-based papers also apply their method to a benchmark example. 34 of the 139 considered papers do not include any application, the rest can be broken down as shown in Figure 4.

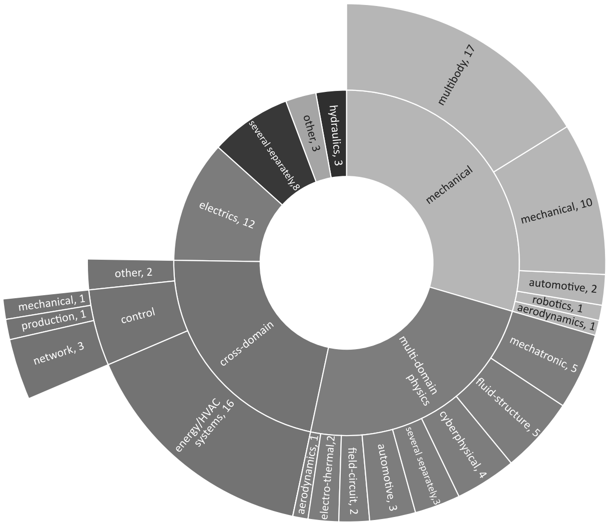

Figure 4: Breakdown of application areas found in the selected literature.

We can observe that the considered applications are mostly physical and almost all cover at least a physical aspect in one partial system. Applications in mechanics constitute the largest share with 30%. Within these, multibody simulations are most common, followed by general mechanical applications. 24% of all papers where an application is found combine two or more physical domains in various manifestations. While applications defined as “cross-domain” (22% of all 105) often also involve multiple physical domains, some non-physical model part is also included – e.g. control strategies or logistics. Summarized as “other” are specific isolated applications such as molecular dynamics or a HIL implementation. The lower-level category “several separately” describes publications where a concept is applied to several examples in different areas separately, not combining them. The sub-category of “multi-domain physics" that is also named “several

separately" likewise comprises several applications in one work but in this case, every separate example couples different domains.
In recent years, purely electrical applications have vanished while cross-domain applications have considerably increased.

5 Model descriptions in the considered literature

Different mathematical model descriptions also require individual solution algorithms and further coupling approaches. In 129 papers, one or more model descriptions out of *ordinary differential equations* ("ODE"), *differential algebraic equations* ("DAE"), *partial differential equations* ("PDE"), *agent based models* ("ABM"), *system dynamics* ("SD"), *synchronous data flow* ("SDF"), *finite element models* ("FEM"), *boundary element models* ("BEM") and *discrete event systems* ("DE") can be identified[1]. ODEs and DAEs are by far the most frequently considered descriptions, as depicted in Figure 5.

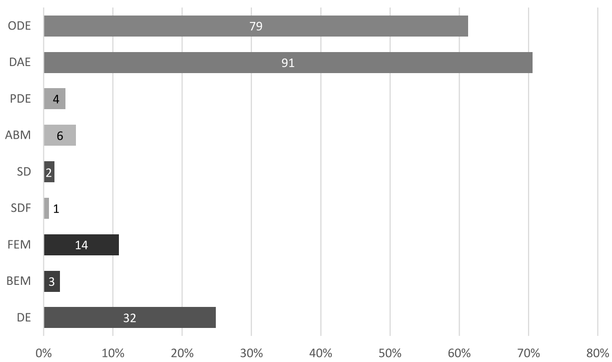

Figure 5: Amount of publications utilizing different kinds of model description.

Over half of the 129 publications (70) cover more than one model description. Thereof, 23 are exclusively dealing with ODEs and DAEs. While ODEs have always been considered in similar measure throughout the "history" told by this selection in research on co-simulation, DAEs have become a more popular topic of interest after the millennium.
33 papers explicitly cover hybrid co-simulation – in the sense of coupling continuous time models with DE systems. Research in this area has increased drastically over time: while none are found in the considered selection before the millennium, nine have been published from 2010-2014 and 21 after 2014. This correlates with the increased occurrence of DE systems.

[1]The two publications on standards have been excepted here since these are not restricted to one or few types of model descriptions.

6 Distinction of algorithms

One major point to be considered when categorizing co-simulation methods are the numerical approaches that come with the nature of the topic. In that respect, the focus can lie on the solution algorithms used by the participating subsystems or the coupling algorithm itself. Prior to the specific algorithms, however, different concepts to approach the intended cooperation can be discerned.

6.1 Distinction by coordination concept

Regarding the general concept of coupling coordination, we can distinguish whether or not an external orchestrator is used to organize the co-simulation. Co-simulations where communication is orchestrated *outside* one of the participating subsystems are classified as using an orchestrator. Without an external orchestrator, data exchange can be handled in one of the subsystems themselves, thus acting as the so-called *master* of one or more other subsystems (*minions*). Note that there are overlaps: If a master or orchestrator coordinates several subsystems while solving an equation, it may be considered as both. Whether or not an external orchestrator is used is defined for 103 papers. A majority (81%) of publications present approaches using an orchestrator. Only 18 publications (17%) do not use an orchestrator, 2 consider both variants. While this trend is also observable in most time frames, publications before the turn of the millennium stand out: within these, more than half do not use an external orchestrator.

6.2 Distinction by interfaces

Although slightly different interpretations for these terms are circulating the literature (cf.[1]), we adopt the term *distinction on "interface level"* from [5] for the division of coupling approaches into *strong* and *loose coupling* methods. *Strong coupling* allows different solvers but requires the same time steps in all subsystems, permanent exchange of coupling data and iteration in every time step while *weak* or *loose coupling* allows different, individual time steps in the partial systems. Whether loose or strong coupling methods are applied can be determined for 115 of the 139 publications. An overwhelming majority of 87% focuses solely on loose coupling algorithms, nine papers (8%) consider only strong coupling methods and five percent cover both approaches. This general observation of a loose coupling dominance is also reflected in most time frames, although strong coupling slightly gains in popularity up to 2009, only to drop again afterwards.

6.3 Distinction by execution sequence

Regarding the sequence in which participating subsimulations are executed, we differ between *parallel* and *sequential* methods. Most commonly, parallel (also called *Jacobi* type) methods are applied. This does not necessarily require de facto computational parallelization on multiple cores, but means that every time data is exchanged, simulation time is the same in every subsystem, so no part can obtain future information on any other system. In sequential (*Gauß-Seidl* type) methods, values at a (in general, one) reference ahead can be used for more accurate approximations in "slower" subsystems. The sequence of execution is defined in 103 of the considered publications.

Over half (55%) of these only investigate or apply parallel methods, almost a quarter exclusively focus on sequential ones and the rest discuss both approaches. The share of (purely) parallel methods gradually increases over time, peaking at 70% in the last five years. This might partly be explained by simultaneous progress in computational parallelization, which can be utilized to speed up coupled simulations, and partly by reasons of implementational simplicity.

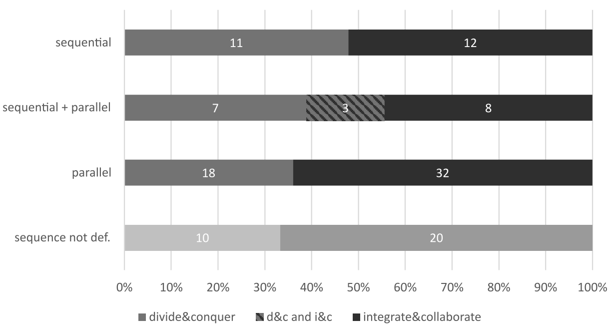

Figure 6: Nexus of sequence of execution and perspective of (de-)coupling.

Figure 6 illustrates the correlation between the sequence of subsystem execution and the perspective concerning the state of development in which the system is partitioned and/or coupled. It is interesting to note that all three publications in which both an integrate-and-collaborate and a divide-and-conquer approach are considered also investigate both parallel and sequential methods. Publications on sequential algorithms show the highest share of those approaching their research question by a divide-and-conquer perspective. Neither property can be made out for six papers. Twelve define the sequence but not the state of development in which the (de-)coupling takes place. The thirty papers with unknown sequence but defined (de-)coupling perspective are included in the statistics, see the lighter colored bar in Figure 6.

6.4 Distinction by iterations

Depending on whether or not the coupling algorithm iterates over its macro steps, we differ between iterative (*implicit*) and non-iterative (*explicit*) approaches with or without a predictor-corrector step. The latter are also called *semi-implicit* as they allow step rejection and thus require rollback even if no iteration as such takes place. Whether the coupling algorithm uses iterations or not can be determined for 112 of the considered publications, whereat this rate has in general – except for the interval 2010-2014 – decreased over time.

In two thirds (74), only non-iterative master algorithms are used. This emphasizes the importance of the development of explicit methods, as – even though implicit approaches surpass explicit ones with respect to accuracy and stability – a majority adheres to explicit methods, be it due to performance reasons, software requirements, or simplicity. Eleven (15%) of the papers using non-iterative methods (amounting to 10% of all 112) apply a predictor-corrector method. A quarter (28) of the total are exclusively using iteration, of which one also applies a predictor-corrector method. Both iterative and non-iterative algorithms are considered in ten papers (8%), predictor-corrector steps in half of these. Figure 7 illustrates that while the research on iterative methods has increased in the years before 2010, it has diminished again in recent years. While this could be interpreted as preference of explicit methods in praxis (as 2000 to 2009 are also the years peaking in a focus on theory), this correlation is disproved by the cross-connection shown in Figure 8. One expectable observation to be made from there is that iterations are not defined in most surveys. Further, in most frameworks, non-iterative methods are applied or it is not defined whether or not iterations (can) take place. Taking a

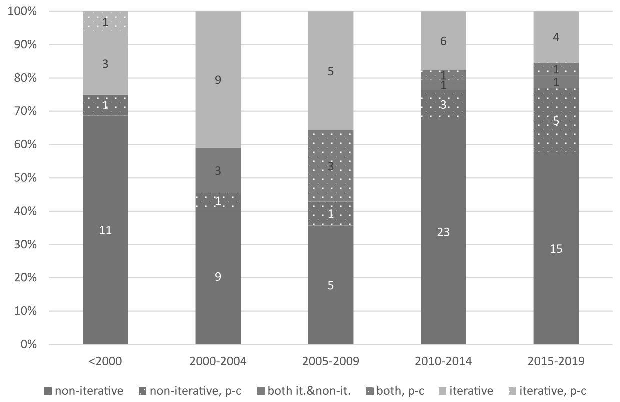

Figure 7: Variation of shares of iterative and non-iterative methods over the years.

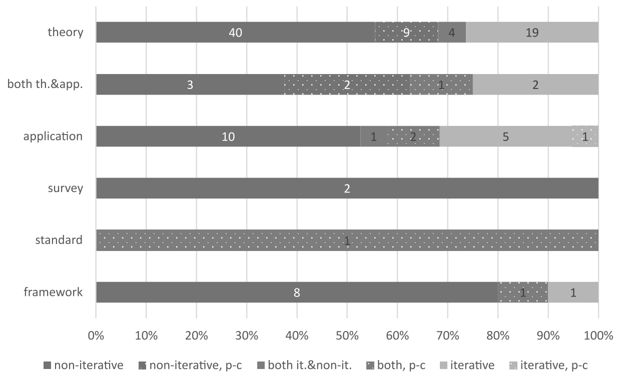

Figure 8: Connection between the main topic of publications and iterations in the master algorithm.

look at the nexus of the sequence of execution and the iterations of master algorithm steps (Figure 9), it is interesting to note that the category of publications exclusively using parallel methods also shows the highest share of non-iterative master algorithms. This implies that those using methods that are more prone to stability issues also do not aim to increase accuracy by iteration. *Both* iterative and non-iterative methods are only considered in publications which also use sequential algorithms (exclusively or in a comparison to parallel ones). There are 23 publications where neither sequence nor iterations are defined and four that define the sequence but do not reveal whether an iterative master algorithm is used.

6.5 Distinction by macro steps

Algorithms with a fixed macro step require all participating simulations to exchange data at previously

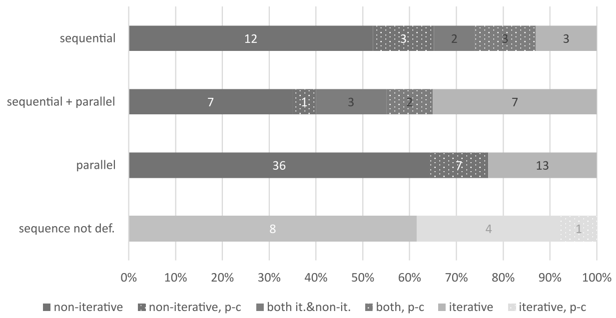

Figure 9: Nexus of sequence of execution and iterations of the master algorithm.

defined synchronization references in – generally – equidistant intervals. Other solutions adapt the macro step in the course of a simulation via step size control, either by step rejection and repetition with a smaller step size if certain tolerances are violated (see f.i. [6]) or by adaption before every macro step execution by predictive error estimation, f.i. via extrapolation or energy residuals [7, 8]. Some loose coupling methods do not necessarily require any synchronized time steps from the sub-simulations apart from the start time, see f.i.[9, 10]. Whether a fixed, adaptive, or individual step size without any simultaneous steps is used is defined for 100 publications. A clear majority of 72% is applying or developing a method with a fixed communication time step. Nevertheless, 35% are using some kind of macro step size control and four present methods allowing no common synchronization reference. There are overlaps, which are shown in detail in Figure 10. While from 2005 on, the amount of publications using fixed or adaptive macro steps does not vary substantially, an exceptional share of those in the category of fixed step sizes stands out in the time frame from 2000-2004. On the other hand, while in all other intervals, methods with fixed macro steps clearly dominate those with adaptive ones, shares are almost equal before 2000. All four works presenting master algorithms that do not require the subsystem solvers to have any step in common are found in the time frame from 2010-2014.

The connection between the kind of master step and utilization of iteration is shown in Figure 11. Not altogether surprisingly, the highest share of adaptive algorithms is found in publications covering non-iterative methods. Apart from the one standard theoretically allowing all three kinds of master steps and iterations (note that the distinction on iterations is not exclusive!), methods that do not require a common synchronization

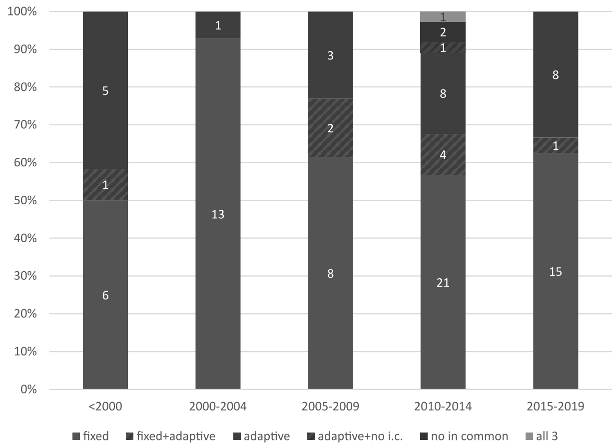

Figure 10: Change in percentages of fixed, adaptive or no common macro step usage in the selected literature over five-year time frames.

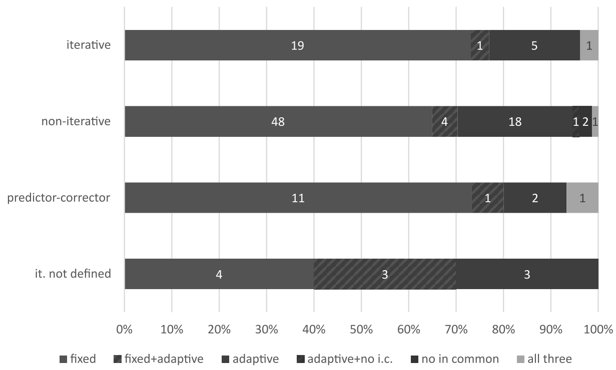

Figure 11: Connection between macro steps and iterations in the considered literature.

step are always non-iterative. In 17 publications, neither if iterations take place nor whether fixed, adaptive, or completely independent step sizes are used has been revealed. 22 did not specify the macro steps, but did clarify whether iterative methods have been used.

7 Distinction by the number of coupled subsystems

While partitions due to latency or activity are sometimes limited to dividing the overall system into two subsystems, further variations in time constants[11] or problems of the integrate-and-collaborate kind [12] often require the cooperation of several or even an arbitrary number of subsystems. The number of co-simulated systems has been defined in all but eight publications, all of which are surveys or formalism descriptions. More than half (56%) of these publications consider exactly two coupled subsystems. Of the remaining 58 papers, 22 are coupling more than two, yet an explicit, integer number of systems. 27% describe methods that can be applied to an arbitrary number of subsystems. Publications that present a theoretical approach for *n* coupled subsystems which have then been tested only on two have also been classified as allowing an arbitrary number of coupled systems.
Figure 12 shows how the different main orientations are partitioned by the number of considered subsystems.

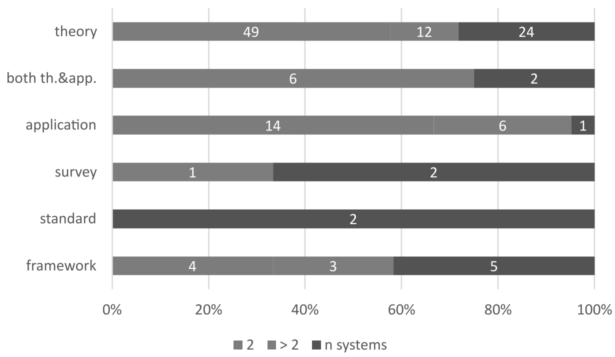

Figure 12: Papers' main emphases partitioned by different numbers of coupled subsystems.

It is not surprising that there are barely publications that focus on an application and still consider an arbitrary number of coupled subsystems. However, all mainly theoretical and applied work is composed predominantly of publications coupling only two systems. Surveys and standards almost always allow an arbitrary number.
The cross-connection of the number of coupled systems and their sequence of execution reveals that the considered literature barely covers investigations of sequential methods for an arbitrary number of systems. This may be explained by the impact of the order of execution in sequential approaches, which gains in possibilities and thus complexity with the number of systems.

7.1 Hierarchical approaches

Furthermore, we characterize whether a hierarchical approach is considered in the literature. The construction of a non-trivial hierarchy, which can increase numerical stability properties [13], is only possible for three or more systems, thus already restricting the selection. Only nine publications decidedly allow hierarchy, four of which describe multirate methods where the integration algorithm itself is partitioned. This illuminates a general lack of existing studies and thus present potential in this area of research.

8 Conclusion

The analysis of the regarded literature has unveiled tendencies towards more popular methods. While most publications cover mainly theoretical aspects, applications are nevertheless manifold and range from mostly physical systems in one or many domains to cross-domain applications including complex controlled systems up to urban scale. Model descriptions are dominated by Differential (Algebraic) Equations but also cover AB or FE models and even DE systems. Hybrid systems, albeit sparsely represented, remain a challenge if approached via coupled simulations as well as they do in a mono-simulation. Non-iterative, parallel loose coupling methods are applied predominantly, even though iterative and sequential approaches entail higher accuracy and better stability properties. Similarly, fixed macro steps are more frequently used than adaptive algorithms, which, as well as the preference to avoid rollback, may be explained by the implementational limitations of commonly known software tools that support co-simulation. The presented analysis has highlighted a lack of research of hierarchical co-simulation, which is addressed in [2].

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