Designing a sound propagation model for a flow rate sensor based on CFD-particle streams

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Abstract

A significant role in almost all industries is nowadays to measure the discharge of gases and fluids, especially in flood prediction and water management. However, a rising error of measured flow rate is often caused by sedimentations, turbulent flow or a deficient space resolution. To overcome those weaknesses an enhanced discharge flow meter will be developed by a combination of specific signal processing algorithms, a new phased array sensor technology and mathematical models to eliminate errors.

Modeling and simulating of the system enables the design of signal processing algorithms related with the new ultrasonic front end, the transmitted signals and the evaluation of the echoes. Therefore, simulation data of a fluid flow model is generated by using a CFD-software package called GAMBIT®/FLUENT®. Based on this CFD-model, particles of streamlines will be integrated into a signal flow diagram of MATLAB®. By realizing a particle-tracking method, a transfer behavior of a piezo-crystal, a sound propagation model for phased array sensors and a sound absorption model as a subsystem, the whole measurement system can be modulated and then simulated.

The aim of this project is to develop a new method to detect the velocity of particles and measure the cross section of open channels permanently with the result to minimize the flow rate error.

1 Introduction

Ultrasonic sensors are widely used in different areas especially in the medical area, in the nondestructive inspection of materials, in automotive parking sensors and in the flow rate measurement of fluids. By using this sensor type as a flow-meter the measurement principle is mainly divided in transit time measurement and in ultrasonic particle reflection measurement. The first one is generally used for clear fluids. The second system however is mainly used for polluted fluids, where the pollution is caused by a bunch of particles or bubbles in the fluid.

In this article a system model for the particle reflection measurement will be discussed. Based on this model a new phased-array ultrasonic sensor, improved signal processing algorithms and adapted mathematical models are generated.

A prerequisite for the particle reflection measurement is a large number of suspended particles or air bubbles in the respective flow; they are considered to move with non-slip in the water [1]. A principle assumption is that the ultrasonic sensor is fixed below the water surface to ensure that a large part of a subaqueous area can be scanned. The concept to capture the whole cross section is realized by phased array technology and illustrated in Fig.1.

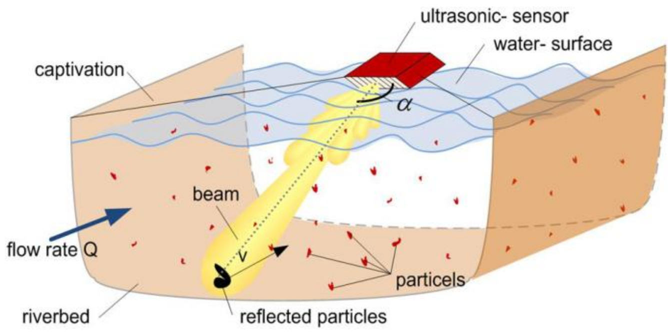

Fig. 1: Fundamental principle of an ultrasonic measurement system in open channels

To detect the discharge in open channels, first of all, the system has to measure the actual contour of the cross section in an open channel or a pipe. This allows accurate and robust calculations of discharge, even if the cross section changes over time due to sedimentation.

For a given velocity vector function $\mathbf{v}$ we can now compute the flux of the water across the water surface A by [2]

$$\underbrace{\text{flux across } A}_{Q} = \iint_A \mathbf{v}\ \mathrm{d}\mathbf{A} \approx \sum_{k=1}^{N} \mathbf{v}(x_k, y_k, z_k)\ \Delta\mathbf{A}_k\ .$$

2 Sound propagation model

One of the key questions in this project is to find an optimal signal processing algorithm chain for the new ultrasonic flow sensor. Therefore, a realistic model for the particle flow and sound propagation has to be developed. On the basis of a signal flow chain, several parts of this system will be discussed in this paper, such as

- defining a CFD-model for pipes,
- presenting a particle-tracking method,
- modeling a transfer behavior of a piezo-crystal,
- describing a sound propagation for phased array sensors and
- setting up a sound absorption model.

The relationship of these parts is illustrated in a signal flow diagram as seen in Fig. 2. The signal flow diagram is used bidirectionally to validate the model. On the one hand the signal flow diagram is used from the transmitted, electrical signal of the sensor $x(t)$ to the reflected sound pressure of the particle $p_{abs}(t)$. On the other hand it is utilized backwards from the reflected sound pressure to the received sensor signal.

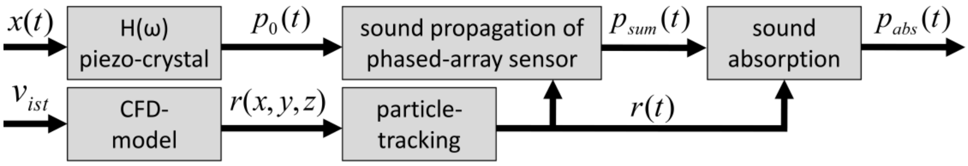

Fig. 2: Signal-flow chain of the sound propagation model

2.1 CFD-model

In this project pre- and post-processing tools are used to simulate a turbulent flow of a pipe or channel, with realistic field conditions. The pre-processing tool GAMBIT® is utilized to design the basic geometries and afterwards to generate a mesh grid. (Fig. 3a) Based on this mesh grid a post-processing tool is used to specify the boundary conditions and generate a turbulent streaming model. This is accomplished with the Computational Fluid Dynamics (CFD) simulation software FLUENT® [3].

To simulate such turbulent streaming models the Navier Stokes equations (equation of momentum, energy equation, continuity equation) have to be solved. For incompressible fluids the equation of continuity is defined by

$$\mathrm{div}\,\mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

as well as the following equation of momentum

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot \mathrm{grad})\mathbf{v}\right) = \rho\,\mathbf{g} - \mathrm{grad}\,p + \eta\,\Delta\mathbf{v}.$$

The variable ρ depicts the density, p the pressure, $\mathbf{v} = [u\,, v\,, w]^T$ the velocity vector, η the dynamic viscosity and $\mathbf{g} = [g\cdot\sin(\delta)\ , 0\,, -g\cdot\cos(\delta)]^T$ the gravity vector with an incline angle of δ.

However, the energy equation is not used to close this mathematical system. By solving those four equations iteratively the laminar flow could be calculated. Nevertheless, in reality a laminar flow does not exist so that this mathematical model has to be enhanced for turbulent flow. An example for such an enhanced model is called the $K - \varepsilon -$Model with two more equations. For the scope of this project the simulation, which is conducted in FLUENT®, uses the Reynolds-Stress-Model because of its ability to simulate an anisotropic turbulence [4, 5]. The simulated velocity profiles for a pipe are illustrated in Fig. 3c.

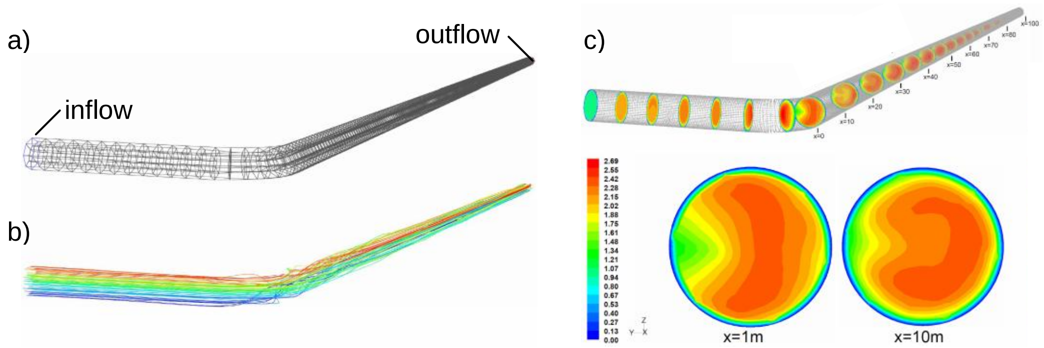

Fig. 3: FLUENT® simulations. a) mesh grid of a pipe contour; b) streamlines of air-particles; c) velocity profile of a simulated pipe (v_{ist}=*1,5m/s*)

Based on this simulation it is possible to generate streamlines for particles in FLUENT® (Fig. 3b). Finally these particle-streamlines can be exported as a time-based-vector-field from FLUENT® and integrated into a MATLAB® data field. Therefore, all streamlines and their particle position with a timestamp are used for further subsystems.

2.2 Particle-Tracking method

By importing the time-based-vector-field of particle positions in MATLAB®, the equidistant spatial resolution in x-, y- and z-direction have to be transformed into an equidistant time-interval. For further simulations the time-interval of particles must be the same as the simulation-interval in MATLAB®. To realize this transformation a spline-interpolation has to be done for each streamline.

To use this particle streamlines on a signal flow diagram and combine them with other time based signals, the sound duration between particle and sensor has to be known. The sound duration t_L is calculated from the particle positions $\mathbf{r} = [x\,, y\,, z]^T \in \mathbb{R}^3$ for each timestamp (n) as seen below:

With the triangle in Fig. 4, the equation is given by

$$(\|\mathbf{r}\|_2)^2 - (\|\mathbf{a}\|_2)^2 - 2 \cdot \|\mathbf{r}\|_2 \cdot \|\mathbf{b}\|_2 \cdot \cos(\gamma_{T1}) + (\|\mathbf{b}\|_2)^2 = 0$$

and the angle γ_{T1} is calculated by

$$\cos(\gamma_{T1}) = \frac{-\mathbf{r}^T\,\mathbf{b}}{|\mathbf{r}| \cdot |\mathbf{b}|}.$$

The particle velocity is computed by the distance of two consecutively particle positions $(\mathbf{r}(n+1) - \mathbf{r}(n))$ and the simulation interval Δt_{sim}.

$$\mathbf{v}_T = \frac{\big(\mathbf{r}(n+1) - \mathbf{r}(n)\big)}{\Delta t_{sim}}$$

In combination with those equations, $\|\mathbf{a}\|_2 = t_L \cdot c_W$, $\|\mathbf{b}\|_2 = t_L \cdot \|\mathbf{v}_T\|_2$ and $\mathbf{v}_T = [v_x, v_y, v_z]^T \in \mathbb{R}^3$ the finally equation is indicated by

$$0 = x^2 + y^2 + z^2 - t_L(2 \cdot \|\mathbf{r}\|_2 \cdot \|\mathbf{v}_T\|_2 \cdot \cos(\gamma_{T1})) \\ + t_L^2(v_x^2 + v_y^2 + v_z^2 - c_W^2) \, .$$

The speed of sound is defined by c_W. To get the sound duration t_L, this equation must be rearranged. Furthermore, the negative solution of the equation has to be ignored.

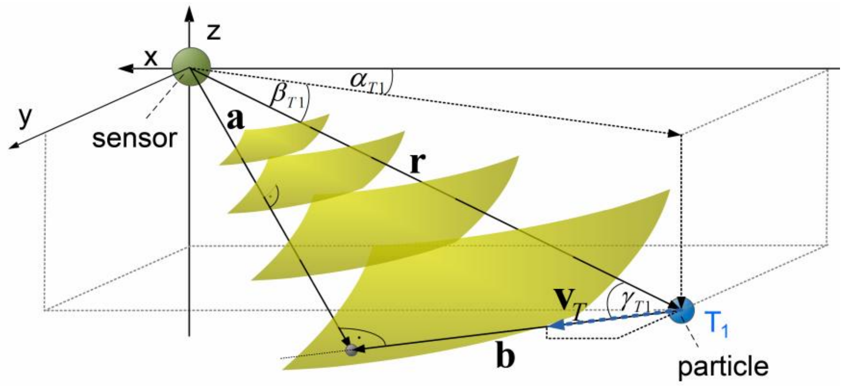

Fig. 4: Geometrical relation of the sound duration and particle position

2.3 Transfer behaviour of piezo-crystal

A characteristic behavior of the measuring system is given by the transfer function of the piezo-crystal. The approach of this paper is to modulate the crystals by initially measuring the frequency responses of the produced crystals. Transforming this frequency response into the time domain leads to the impulse response of the piezo-crystal, which is specified with $p_0(n)$. To approximate this measured impulse response to an N-length FIR-filter [6, 7] with an output signal of $\hat{p}_0(n)$ and filter coefficients of $\mathbf{b} \in \mathbb{R}^N$

$$\hat{p}_0(n) = \mathbf{b}^T\mathbf{x}(n-1) = \sum_{k=0}^{N} b_k \cdot x(n-k-1)$$

the error of both signals $e(n) = p_0(n) - \hat{p}_0(n)$ have to be minimized. An option to minimize this error and to find the optimal filter coefficients requires first of all the definition of the mean square error (MSE).

$$MSE = E\{e^2(n)\} = E\{[p_0(n) - \hat{p}_0(n)]^2\} = E\{[p_0(n) - \mathbf{b}^T\mathbf{x}(n-1)]^2\} \\ = E\{{p_0}^2(n)\} - 2\mathbf{b}^T E\{\mathbf{r}_{xp}(-1)\} + E\{\mathbf{b}^T\mathbf{R}_{xx}\mathbf{b}\} \, .$$

The autocorrelation matrix of filter input signals $\mathbf{x}(n-1)$ is defined by $\mathbf{R}_{xx}$ and has a symmetrical toplitz structure. The cross correlation vector $\mathbf{r}_{xp}$ describes the relationship between the input signal and the expected impulse response of the piezo-crystal $p_0(n)$. Minimizing the MSE-equation with the use of partial derivative reveals

$$\nabla_b E\{e^2(n)\} = \mathbf{R}_{xx}\,\mathbf{b} + \mathbf{r}_{xp}(-1) \stackrel{!}{=} \mathbf{0}$$
$$\mathbf{b}_{opt} = \mathbf{R}_{xx}^{-1}\ \mathbf{r}_{xp}(-1)\,.$$

By solving this equation iteratively with a Levinson-Durbin algorithm the optimal filter coefficients for the FIR-filter was found [8, 9, 10]. Transforming the impulse response of the filter backwards into the frequency domain and comparing them with the measured piezo-crystal leads to Fig. 5. This figure illustrates the measured and approximated frequency response of a FIR- filter with N =65 coefficients and a sample frequency of 10MHz.

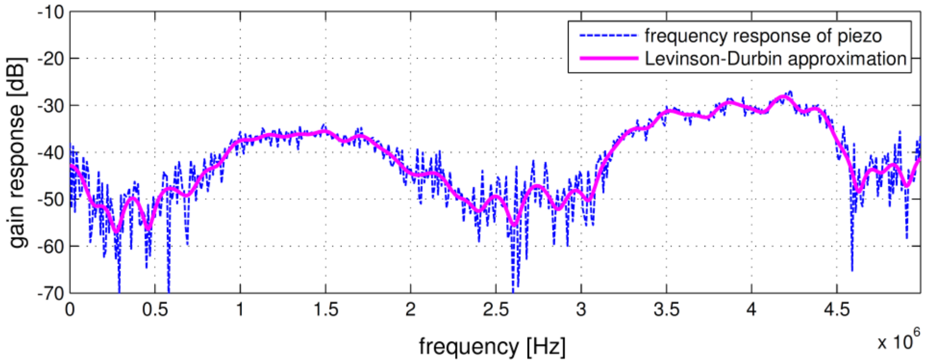

Fig. 5: Approximated frequency response of the FIR-filter and measured piezo-crystal

2.4 Sound propagation of phased array sensors

As illustrated at the beginning of this paper an ultrasonic phased array sensor is used to detect a bunch of particles across a channel or a pipe cross section. Therefore, the ultrasonic sound propagation for this sensor type has to be analyzed in more detail. Basically the sound propagation of phased array sensors is characterized by a point sound source on the one hand and a radiation pattern on the other hand. For the acoustic fare field the pressure p_n of a single point sound source, in a defined particle distance $\|\mathbf{r}\|_2$, is given by [11, 12]

$$p_n = \frac{j\,\omega\,A\,p_0}{4\,\pi\,c_W\,\|\mathbf{r}\|_2}\ \mathrm{e}^{-j\,k\,\|\mathbf{r}\|_2}\,.$$

In that equation k is equal to $2\pi f/c_W$ and means the angular repetency, A is equivalent to surface of the source and c_W to the speed of sound. Pressure p_0 is the output signal of the FIR-filter in the upper chaper.

To focus the power of the beam in a specific direction, every single point sound sources (radiators) have to be controlled by a phase shift. In a first step this concept of phased array processing can be modulated by the Huygens' superposition principle [13]. Fig. 5a illustrates an angular-dependent wave front caused by the phase-shifted signal of each single radiator.

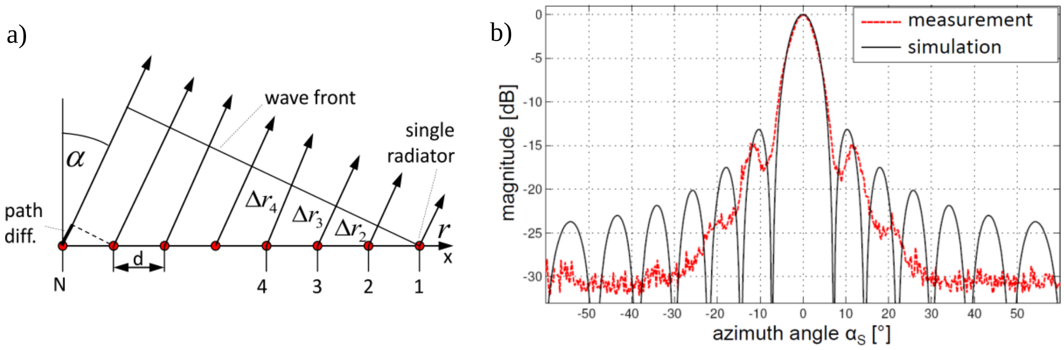

Fig. 6: a) 1 dimension of a linear phased array in azimuth direction; b) radiation pattern

The pressure of a linear array with N elements in azimuth direction, an element-pitch of $d = \lambda/2$ and a path difference of $\Delta r_n = (n-1)\, d \sin(\alpha_S)$ are described with the equation:

$$p_{sum} = p_n \cdot \sum_{n=0}^{N} \mathrm{e}^{-j\,k\,(n-1)\,d\,\sin(\alpha_S)}$$
$$= \frac{j\,\omega\,A\,p_0}{4\,\pi\,c_W\,\|\mathbf{r}\|_2}\;\mathrm{e}^{-jk\,\|\mathbf{r}\|_2} \cdot \sum_{n=0}^{N} \mathrm{e}^{-jk\,(n-1)\,d\,\sin(\alpha_S)}$$

p_{sum} describes the pressure of the radiation pattern, n describes the number of elements, p_n the single point sound source and α_S the azimuth angle of the main lobe at which the sensor shall be pointed Fig. 6b. With the extension of the particle position in chapter "particle tracking method" and an elevation angle of the radiation pattern the complete equation is given by

$$p_{sum} = \frac{j\,\omega\,A\,p_0}{4\,\pi\,c_W\,\|\mathbf{r}\|_2}\;\mathrm{e}^{-jk\,\|\mathbf{r}\|_2} \cdot \sum_{n=0}^{N} \mathrm{e}^{-jk\,(n-1)\,d\,\sin(\alpha_S-\alpha_{T1})} \cdot \sum_{m=0}^{M} \mathrm{e}^{-jk\,(m-1)\,d\,\sin(\beta_S-\beta_{T1})}$$

The sensor angle in azimuth direction is equivalent to α_S and in elevation direction β_S. Similar to this are the particle angles with α_{T1} and β_{T1}. The number of elements in elevation direction is specified with M.

2.5 Acoustical absorption:

The last part of the model is characterized by the acoustical absorption and depends on the mixture of the fluid. Based on atomic movement, caused by the sound propagation, thermal energy gets lost. In this case the absorption can be defined by an exponential decay of amplitudes with an increasing distance. For the classical, single atomic absorption model the coefficients are consist of a thermal conductivity coefficient $\propto_t$ and a coefficient of viscosity $\propto_v$ [12]. Both coefficients can be calculated by

$$\propto_t = \frac{\kappa - 1\eta}{2\,\kappa} \frac{\nu\,\omega^2}{\rho\,c_v\,c_w^3}$$

$$\propto_v = \frac{2\,\eta\,\omega^2}{3\,\rho\,c_w^3}$$

where η depicts the constant of viscosity, ν the thermal conductivity, ρ the density, c_w the speed of sound, κ the adiabatic exponent and ω the angular frequency.

Due to the atomic complexity of a natural fluid, an empirical measurement has to be conducted. Therefore, further approximation coefficients K_m and $\propto_m$ are used to optimize the acoustical absorption model [14]. All in all, the complete absorption model with the pressure of sound propagation p_{sum} is given by

$$p_{abs} = p_{sum}\,K_m\;\mathrm{e}^{-\|\mathbf{r}\|_2\,(\alpha_t+\alpha_v+\alpha_m)}.$$

The absolute pressure for the theoretical single atomic model, the measurement and the approximated model is illustrated in Fig. 7 for each distance.

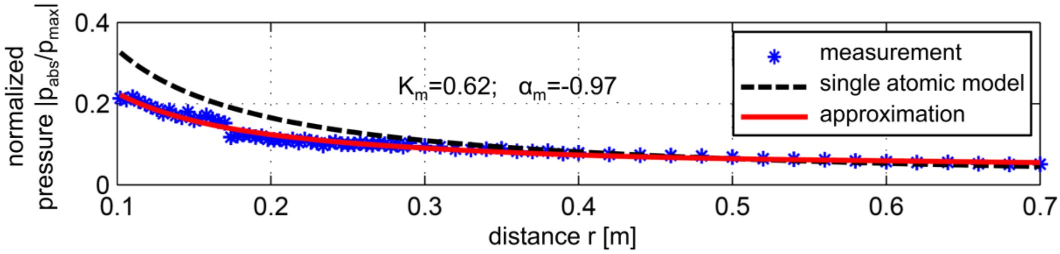

Fig. 7: Absolute pressure of the acoustical absorption

3 Validation of the model

For the verification of the whole model a discharge test bench was constructed Fig. 8a. By using this test bench a defined discharge of water and air bubbles can be circulated in an acoustic penetrable pipe. Measuring the discharge of bubbles along the pipe with the modulated ultrasonic phased array sensor in a full water tank, signals can be received.

Similar to this construction a CFD simulation model with the same specifications, dimensions and boundary conditions was developed, see Fig. 8b. Together with the described signal flow chain on the top and the identically transmitted signals of the test bench, the received signals can be generated.

To compare both received signals (from the test bench and the model) with each other an established cross-correlation-algorithm from the company NIVUS is used [15]. This algorithm is analyzing the signals of the reflected air bubbles and their respective velocity. Both results are illustrated in Fig. 9.

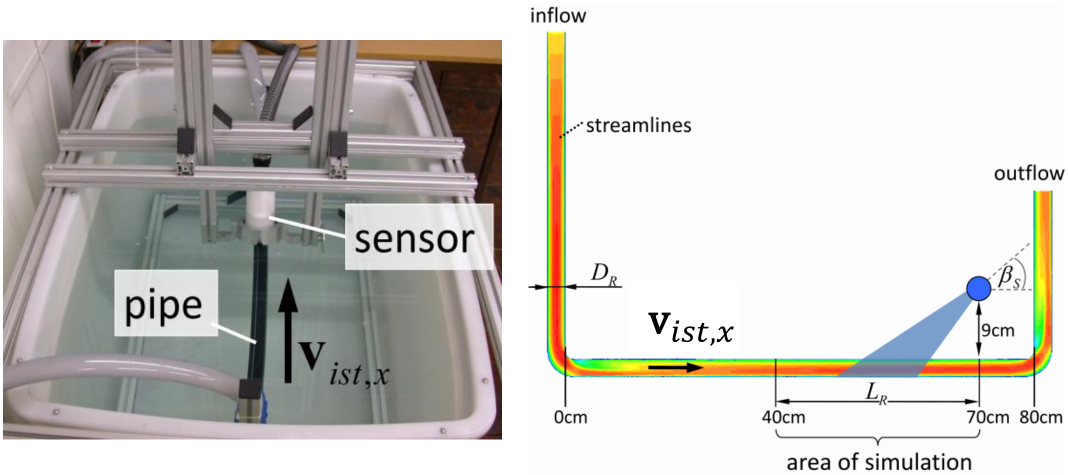

Fig. 8: a) discharge test bench on the left; b) CFD-model on the right

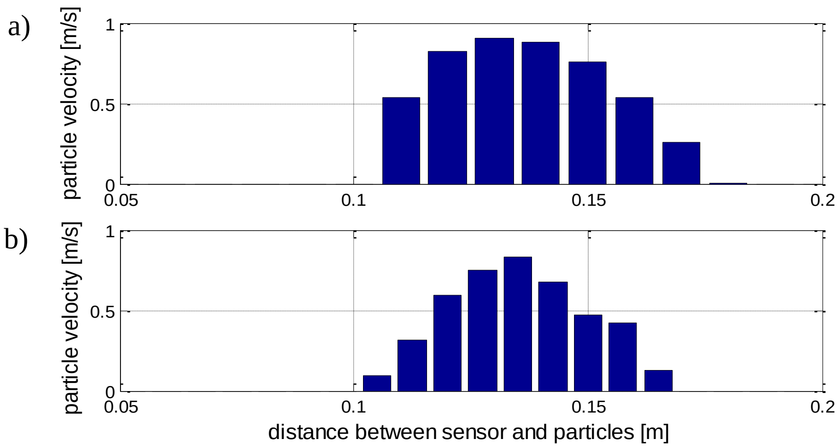

Fig. 9: a) measured velocity in sensor direction; b) simulated discharge; $\mathbf{v}_{ist,x} = 1{,}5\,\frac{m}{s}$

4 Conclusion

All in all, the whole system of ultrasonic particle reflection measurement as well as the natural fluid flow are modulated in this paper. With the help of the CFD-software GAMBIT®/FLUENT® the fluid flow is modulated and afterwards simulated. By integrating the simulated fluid flow into a MATLAB®-based signal flow diagram, further mathematical models from the flow rate sensor are simulated and presented. The verification of the MATLAB® models with the measurements are shown in this paper for the transfer behavior of a piezo-crystal (Fig. 5), the sound propagation of a phased array sensor (Fig. 6b) and the sound absorption (Fig. 7). To validate the model the cross-correlation algorithm is used to compare the received signals from the test bench with the simulated signals.

With the help of this modulation an optimal algorithm will be found in order to detect the velocity vector as well as the contour of the cross section. This is the basis to identify the specifications of the hardware components, which will be used to build the first prototype.

5 References

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Presenting Author's Biography

Manuel Haide studied at the University of Applied Sciences in Ulm. He graduated with a German degree in Industrial Electronics and a Master of System Engineering. Presently he is a member of the research staff at the Institute for Applied Research in Ulm. The focus of his work is to specify and analyze different signal processing methods for a new ultrasonic phased array system.

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