

Improving Quantum Computer Simulations

Binh Trieu, Guido Arnold, Marcus Richter

Quantum Bits

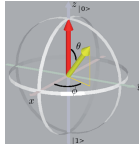
Classical Bit

0

1

0 or 1

Quantum Bit



$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle \Leftrightarrow \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

Quantum Operations

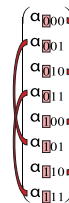
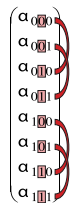
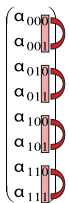


$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

H_0

H_1

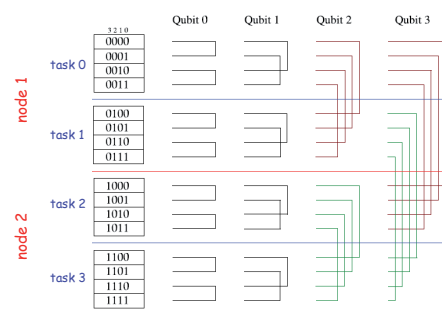
H_2



Need for Simulation

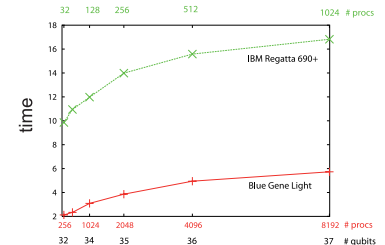
- Develop quantum algorithms
- Analyze robustness to decoherence and gate errors
- Test scalability of error correction codes
- Optimize pulse sequences for ion trap quantum computers

Communication scheme



Scaling Properties

Highly efficient up to 37 qubits using 1k (8k) Procs and 3 TB Memory



Quantum Error Model

Gate imperfections

Rotations $R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Phase shifts $P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

- Decompose e.g. $H = R(\pi/4)P(\pi)$

- Introduce errors $\theta' = \theta + \epsilon_\theta$
 $\phi' = \phi + \epsilon_\phi$
gaussian distributed with standard deviation σ

Decoherence

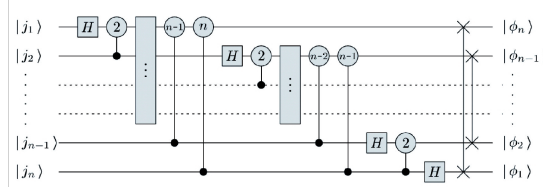
Apply with probability $p/3$ each:

- Bit-flip $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

- Phase-flip $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Both $-i\sigma_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Application: Robustness of the Quantum Fourier Transform



• Errornorm $e^2(\sigma, p) = |\psi - \psi_{exact}|^2$ reveals threshold at $\sigma \approx 10^{-2}$ (more robust than Grover)

• Dependence of the decoherence error on the system size: $n \rightarrow 2n \Rightarrow p \rightarrow 1/10 p$

• Visualisation of Qubit measurement:

