Fuzzy Arithmetic for the Finite Element Modeling of Structures in the Presence of Uncertainty

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Abstract

A problem often encountered in the numerical simulation of real-world systems consists of the fact that exact values for the parameters of model equations are used, even though the parameter values exhibit a rather high degree of uncertainty in reality. A good way of integrating uncertainties in the simulation of these real-world systems is the fuzzy arithmetical approach implemented in the transformation method. The approach will be applied to uncertainties in mass positioning and the support of a simplified automotive component. Its dynamic behavior is investigated.

1 Fuzzy numbers and the transformation method

To include uncertainties in the simulation of real-world systems, the fuzzy arithmetical approach is used: the uncertain parameters are described by fuzzy numbers in contrast to random numbers used in stochastic approaches. A fuzzy number is a special case of a fuzzy set and appears to be an ideal way of describing uncertain parameters.

To be able to work with uncertain parameters described by fuzzy numbers, the transformation method, developed by Hanss [1], can be used. This method reduces fuzzy arithmetic to regular crisp-number arithmetic via a well-defined decomposition-recomposition scheme. It ensures that the complete information about the uncertainties is included in the model: The uncertain parameters are discretized, then transformed, and the resulting parameter variation sets are computed in a pre-processing step. The problem is then evaluated for each parameter set by a commercial finite element (FE) code leading to a fuzzy-valued output as the result of the numerical simulation. Due to the specific properties of the transformation method, the method can be coupled to any existing finite element code [2,3]. The post-processing step includes the visualization and analysis of the fuzzy FE simulations: the transformation method allows each uncertain model parameter to be rated with respect to its particular influence on the overall uncertainty of the output, resulting in a fuzzy arithmetical sensitivity analysis.

In the following, an example on the practical application of the transformation method is shown. The example will simulate and analyze the vibration behavior of an automotive component, i.e. a circuit board from a state-of-the-art engine control unit. The circuit board is simplified as a thin undamped plate clamped on two sides taking into account its real assembling state in the engine hood of the car. The effects of uncertainties in the positioning of two masses on this plate as well as uncertainties in the clamping of the plate, simulating the state of 'nearly perfectly clamped', are investigated. Results for a combination of both phenomena will be mentioned. The uncertain parameters ***f*** or fuzzy output ***q*** of the system are displayed in bold characters.

2 Model and Scenarios

The circuit board under investigation is modeled in a simplified way as an undamped plate clamped on two sides (Figure 1). This is done in reference of the real assembling situation where the board is fixed in slots on both sides with a thin layer of rubber between the unit and the slot. The FE model consists of standard shell elements. The dimensions of the plate are given in Figure 1, the material data is given in Table 1.

Thickness $t = 1$ mm	Poisson's ratio $\nu = 0.494$
Young's modulus $E = 2.82\text{e}10$ kg/(ms^2)	Density $\rho = 2420$ kg/m^3

Table 1: Material data of the investigated plate.

In an approximation of the various different types and sizes of electronic components located on such a board, two planar masses are added, each mass 5 times the mass of the original area of the plate it covers. As the positioning of the electronic components varies between different models of the circuit board, the effect of their actual position on the board (Figure 1) on the dynamic behavior is investigated. The plate is perfectly clamped in this case.

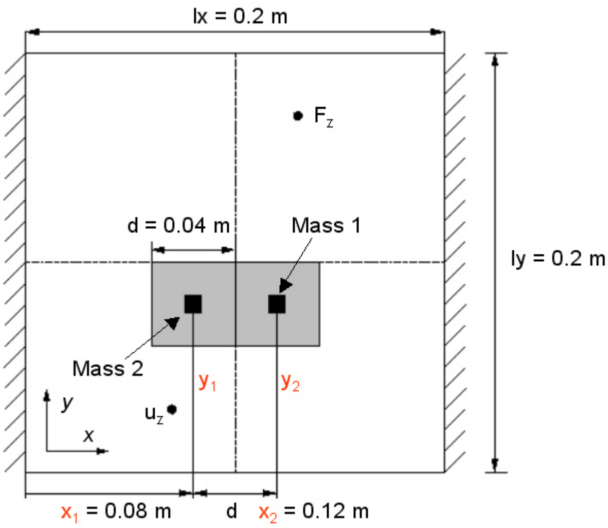

Figure 1: Top view of the plate with the range of uncertainty in the positioning of the two masses (grey), clamped on two sides.

In a second numerical experiment, an uncertain support of the plate with fixed masses is the area on interest. The goal is to simulate a *nearly* perfectly clamped plate, which complies much better with the real assembling situation. Instead of a fixed degree of freedom in translational *z*-direction and the rotational degree of freedom about the *y*-axis, linear springs are added on the plate edges in these directions (Figure 2). By assuming the two constants of the springs, $\boldsymbol{k}_{uz}$ and $\boldsymbol{k}_{roty}$, as uncertain parameters of the system, the desired effect is obtained.

A third numerical scenario consists of a combination of the former two with variations in the positioning of the masses and an added uncertainty in the support of the plate.

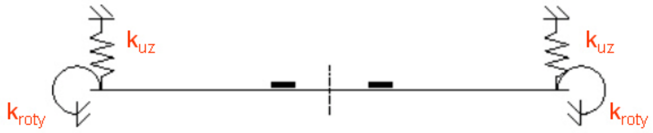

Figure 2: Side view of the plate: linear springs in *z*-direction/about *y*-axis with spring constants $\boldsymbol{k}_{uz}$ and $\boldsymbol{k}_{roty}$ simulate support uncertainties.

The frequency response function (FRF) of the system in a range from 0 to 1000 Hz is under investigation. The plate is excited by a time-harmonic force acting in *z*-direction at point $F_{z.}$ The uncertain output $\boldsymbol{q}$ of the system is the amplitude characteristic of the frequency response of the plate with respect to the locations F_z and $u_{z.}$ A combination of optiSLang, Matlab, and ANSYS is used for the fuzzy FE simulations. All scenarios and data on the uncertain parameters are given below (range of the fuzzy number $\boldsymbol{x}$: (lower bound, $\mu(\boldsymbol{x})$=0), mean value, $\mu(\boldsymbol{x})$=1, (upper bound, $\mu(\boldsymbol{x})$=0)):

- **Scenario 1**: 4 uncertain parameters in the *x*- and *y*-coordinates of the positioning of mass 1 ($\boldsymbol{x}_1$,$\boldsymbol{y}_1$) and mass 2 ($\boldsymbol{x}_2$,$\boldsymbol{y}_2$):
  $\boldsymbol{x}_1$ = (0.06) 0.08 (0.1) m = $\boldsymbol{y}_1 = \boldsymbol{y}_2$ (symmetric, triangular fuzzy number),
  $\boldsymbol{x}_2$ = (0.1) 0.12 (0.14) m (symmetric, triangular fuzzy number),
  general (*m* = 3) and reduced (*m* = 5) transformation method.
- **Scenario 2**: 2 uncertain parameters representing uncertain clamping, i.e. spring constants $\boldsymbol{k}_{uz}$ and $\boldsymbol{k}_{roty}$:
  $\boldsymbol{k}_{uz} = (10^4)\ 10^{4.32}\ (10^{4.5})$ N/m (symmetric, triangular fuzzy number),
  $\boldsymbol{k}_{roty} = (10^{0.1})\ 10^{1.2}\ (10^{1.5})$ N/m (symmetric, triangular fuzzy number),
  general (*m* = 10) and reduced (*m* = 10) transformation method.
- **Scenario 3**: combination of scenarios 1 and 2: 6 uncertain parameters:
  4 ***x***- and ***y***-coordinates position of mass 1 and mass 2
  2 spring constants $\boldsymbol{k}_{uz}$ and $\boldsymbol{k}_{roty}$.
  reduced transformation method (*m* = 10).

3 Results

Results of Scenario 1

Figure 3 shows the uncertain frequency response output of scenario 1. The general transformation method is applied with the decomposition number *m* = 3, resulting in N = 337 parameter variations and, thus, system evaluations. The red line reflects the result for the membership level $\mu(\boldsymbol{x})$= 1, the black line for $\mu(\boldsymbol{x})$ = 0, the dashed lines for $0 < \mu(\boldsymbol{x}) < 1$. Imagine the axis of membership level $\mu(\boldsymbol{x})$ pointing out of the plane and the figure being a projection of the fuzzy FRF on this plane.

There is an overall increase of the amplitude bandwidth around the mean value FRF ($\mu(\mathbf{x})= 1$). However, the relative increase is not the same for each of the eigenfrequencies, nor is it proportional to the frequency value. In fact, there is a relation to the mode shape of the plate at that frequency. The absolute measure of influence κ [1] (not plotted here) shows a rapid increase on the overall uncertainty of the system around the eigenfrequencies.

Depending on the mode shape, the positioning of the masses affects the dynamic behavior of the plate either significantly or hardly at all. That can be seen at the following three examples: The mode shape of the eigenfrequency at about f = 320 Hz is symmetric to the main symmetry axis in y-direction. Remembering that the clamping is parallel to the y-axis, it can be concluded that the uncertainty of the position of both masses does not have a significant influence on the amplitude of the system. In the opposite case, there is the behaviour of the plate at about f = 360 Hz: this mode shape is more prone to a varying position of the masses due to an added symmetry about the x-axis. Between f = 700 Hz and f = 800 Hz there are two mode shapes that are very similar in form. Note how the method struggles with the refinement of the response function especially for higher values of the displacement as the mode changes its shape. The relative measure of influence ρ [1] (not plotted here) underlines these observations.

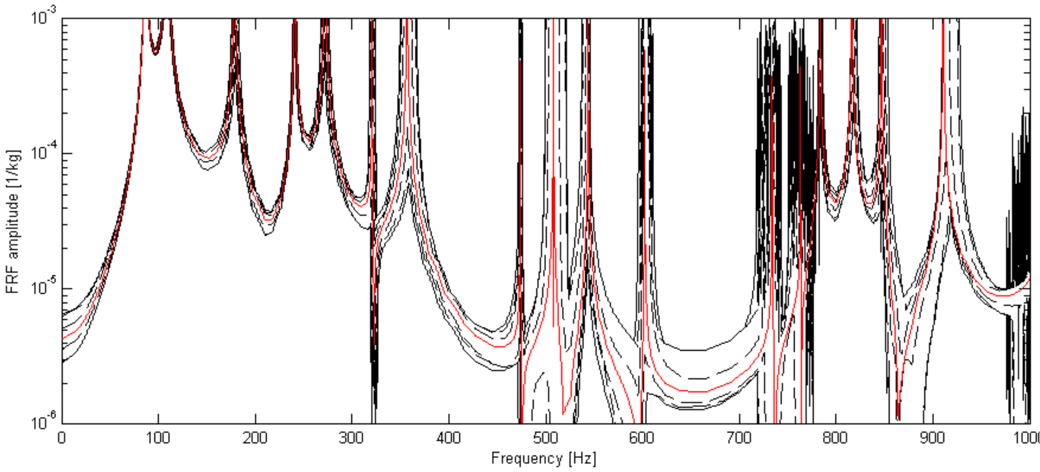

Figure 3: Frequency response of the plate, scenario 1.

The application of the classification criterion [1] reveals that there is a non-monotonic influence of the input parameters for higher frequencies, so the general transformation method should be applied. Nonetheless, a simulation for comparison can be made with the reduced transformation method and the decomposition number m = 5, which leads to a substantially smaller number of parameter variations (N=51) to be evaluated. As a result of this comparison, it can be stated that the reduced method leads to a narrowed uncertainty range and that the results are satisfactory for a first estimation of the system, highly motivated by the smaller number of evaluations needed.

Results of Scenario 2

Figure 4 shows the uncertain frequency response output of scenario 2. The general transformation method was applied with the decomposition number m = 10, resulting in N = 650 parameter variations and, thus, system evaluations. The red line reflects the result for the membership level $\mu(\boldsymbol{x})$= 1, the black line for $\mu(\boldsymbol{x})$ = 0, the dashed lines for $0 < \mu(\boldsymbol{x}) < 1$.

The plate is simulated as being nearly perfectly clamped in this scenario. Even though the uncertain input parameters of the system are symmetric, the increase of amplitude bandwidth is non-symmetric with respect to the mean value FRF. The system is very sensitive to a change in the support, especially to a loosening of the clamping, i.e. to a reduction of the spring constants $\boldsymbol{k}_{\mathrm{uz}}$ and $\boldsymbol{k}_{\mathrm{roty}}$. It can also be noted that even with such a small variation in the support, it is very hard to identify a frequency f > 400 Hz safe for operating the plate in this scenario.

The relative measure of influence ρ (not plotted here) shows us that the influence of the spring constant $\boldsymbol{k}_{\mathrm{roty}}$ is much higher than the translational spring constant $\boldsymbol{k}_{\mathrm{uz}}$ in the lower frequency range (f < 300 Hz). Its influence gradually decreases as that of the spring constant $\boldsymbol{k}_{\mathrm{uz}}$ increases (remember, the sum of all ρ is always equal to unity). A reason for this is that as the mode shapes exhibit more complicated forms, the translational springs are stressed more than the rotational ones.

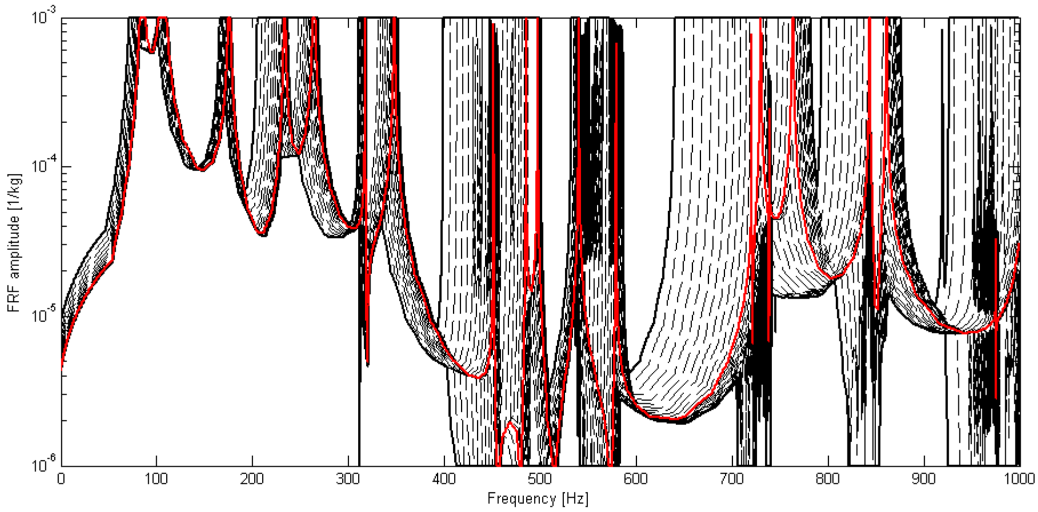

Figure 5: Frequency response of the plate, scenario 2.

The application of the classification criterion for this problem indicates the non-monotonic influence of both uncertain input parameters, even though a simulation using the reduced transformation method is performed, leading to results comparable in quality to the ones mentioned for scenario 1.

The results of scenario 3 using 6 uncertain parameters in both the positioning of the masses as well as the uncertainty in the support of the plate are carried out by applying the reduced transformation method and a decomposition number of $m = 10$. The reason for that is the higher number of uncertain input parameters. Even though all parameters behave non-monotonic with increasing frequency f, a compromise between exactness of the result and computational effort can thus be achieved. The result (not plotted here) is a combination of scenarios 1 and 2. The uncertainty in the clamping of the plate dominates the overall behavior of the system while the uncertainty in the positioning of the masses affects the widening of the reaction range to higher frequencies.

4 Conclusions

Numerical experiments on a plate have been performed with the purpose of investigating the effect of uncertainties in the positioning of planar masses on the dynamic behavior of the plate by examining the frequency response of the system. A second run of experiments has studied uncertainties in the support of the plate by loosening its clamping. A final combination of both effects showed a clear predominance of the support problem.

5 Acknowledgments

The financial support by the Robert Bosch GmbH, Stuttgart, Germany, is gratefully acknowledged.

6 Literatur

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