Object-oriented dynamics modeling for simulation, optimization and control of walking robots

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Abstract

This paper shows how efficient object oriented dynamics modeling and optimal control techniques form the basis for generating optimal gaits for four-legged and biped robots. Dynamics of walking robots, dynamics algorithms and our object-oriented approach are shortly repeated as well as optimal control techniques. Numerical and experimental results for a four-legged and a humanoid robot are cited.

1 Introduction

The development of flexible, dynamic and autonomous (i.e. guided by external and internal sensor data) motions for quadrupedal and humanoid robots is still a challenging and mainly unsolved task. The mechanics of legged robots is characterized by a free-floating, tree-like multibody systems structure with a high number of degrees of freedom consisting of many links and many actuated joints. Due to the frequent changes of the contact situation during legged locomotion frequent changes in the dynamic model occur. This special structure can usually not be utilized for an efficient implementation when using general purpose multibody systems (MBS) formalisms and tools. To cope with the high complexity of the nonlinear dynamics of legged robots model-based methods for real-time actuator control, for trajectory optimization, and for controller design specifically tailored to the *one* scenario of legged robots must be developed. For rapid and virtual prototyping a most desirable goal is not only to apply the same abstract dynamic model representations and mathematical models but also as many parts as possible of the same *code* for off- and on-line evaluations of legged robot dynamics during *all* stages of design, development, implementation and operation of a legged robot. To facilitate the investigation of new concepts of nonlinear model-based optimization and control methods also the sensitivities of the legged robot dynamics model with respect to its state variables and parameters are needed. The formalisms and tools applied at the same time have to cope with (i) the complex underlying mechanical model, characterized by many degrees of freedom, actuator dynamics, and interaction of the robot with its physical environment including time-varying contacts and collisions, (ii) the wide range of required numerical schemes, including kinematics, dynamics, sensitivity information, etc., (iii) efficient code generation which is particularly vital for on-line computations, and must rely on the power of dedicated (recursive) algorithms, and (iv) interaction of the computational model with the system it is running on, e.g., communication with sensors and actuators.

2 Dynamics of Walking Robots

Legged robots are described as free-flying, fully three-dimensional rigid MBS experiencing contact forces to allow for the modeling of walking or running gaits including phases with all legs in a flight phase and for the application of various ground contact models, though actually feet-ground interaction is modelled by completely inelastic collision.

2.1 Forward dynamics model

The joint space equations of motion of a rigid multibody system in the presence of ground contact, i.e. tip contact forces and holonomic tip constraints, are

$$\begin{aligned} \ddot{\mathbf{q}} &= \mathcal{M}^{-1}(\mathbf{q})\left(B\mathbf{u} - \mathcal{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{G}(\mathbf{q}) + J_c^T(\mathbf{q})\,\mathbf{f}_c\right) && (1) \\ 0 &= \mathbf{g}_c(\mathbf{q}), && (2) \end{aligned}$$

where $\mathbf{q}, \dot{\mathbf{q}} \in \mathbb{R}^{n_d}$ are the column matrices of joint position and velocity variables, $\mathcal{M}$ is the positive-definite joint space mass matrix, $\mathcal{C}$ and $\mathcal{G}$ are the vectors of gyroscopic and gravitational forces, the vector $\mathbf{u} \in \mathbb{R}^m$ is the vector of actively controlled joints, which is mapped with the constant matrix B. The n_c holonomic ground contact constraints $\mathbf{g}_c \in \mathbb{R}^{n_c}$ result in a constraint Jacobian $J_c = \frac{\partial \mathbf{g}_c}{\partial \mathbf{q}} \in \mathbb{R}^{n_c \times n_d}$ and $\mathbf{f}_c \in \mathbb{R}^{n_c}$ is the vector of ground constraint forces.

2.2 Sensitivity computations

Sensitivity computations are essential in problems involving optimization, non-linear analysis, and linearization. Linearized forward and inverse dynamics models for robots are useful in motion planning and control applications [JR93]. The objectives in trajectory optimization of legged robots to apply sensitivities of the forward dynamics model are (i) more robust and faster convergence in gradient based methods and (ii) more reliable approximation of the Hessian from the exact analytical first derivatives. In contrast to numerical differentiation this technique is robust, avoids errors, and is scalable to large-dimensional systems such as full three dimensional humanoid models. For trajectory optimization the sensitivity of the state space forward dynamics model (1) w. r. t. control and state variables is required:

$$\delta\ddot{\mathbf{q}} = \nabla_{\mathbf{u}}\ddot{\mathbf{q}}\,\delta\mathbf{u} + \nabla_{\mathbf{q}}\ddot{\mathbf{q}}\,\delta\mathbf{q} + \nabla_{\dot{\mathbf{q}}}\ddot{\mathbf{q}}\,\delta\dot{\mathbf{q}}. \tag{3}$$

2.3 Dynamics algorithms

The numerical evaluation of forward dynamics is performed in terms of $\mathcal{O}(N)$ Articulated Body Algorithm (ABA) [Fea83]. This is an efficient and numerically stable algorithm for moderately constrained tree-structured systems with many degrees of freedom. The contact constraint forces and reduced dynamics are calculated using methods described in [Har99] applying the operational space inertia matrix [Kha83]. This inertia matrix serves as a basis for the solution of collision dynamics, too. It is used to calculate the amount of impulse when tips of the robot collide with the ground. The method to distribute the impulse over the MBS is described in [AKD94]. The algorithms applied to calculate the linearized forward dynamics model are based on differentiation of a recursive symbolic forward model [JR93]. The numerical evaluation of $\delta\ddot{\mathbf{q}}$ is of order $\mathcal{O}(N)$. All algorithms are of recursive nature and well-suited for efficient object-oriented implementation.

2.4 Object oriented approach

An object oriented implementation of the dynamics algorithms has been established. The approach consists of a carefully designed class hierarchy which separates the specification of the mechanical model and the desired calculations, referred to as specification model, from the implementation part. The latter consists of algorithm-specific code-generators (Builder) creating executable instances (Solver) finally performing the desired multibody computations. The two main advantages for the usage of Builder/Solver components are on the one hand the application of dedicated efficient problem-specific algorithms and closed-form solutions to reap maximum efficiency and not to rely on one general purpose formalism and to allow for flexible algorithm interchange. On the other hand an object-oriented design using well-defined interfaces and classes allows for modeling numerical algorithms and complex relations between them, which reflect mathematical interdependencies, often not possible to be represented by pure symbolic approaches. This results in an efficient and light-weight code without a compile-to-code step required and for re-use of code and numerical results during run-time. On the other hand the specification model permits more general topologies, components, and algorithms, indispensible for further research which will be directed to legged robot models of increased complexity, including arm dynamics and advanced actuator concepts using artificial muscles. More details may be found in [HSvS04].

3 Optimal Control Techniques

3.1 Optimal control problem

Finding stable gaits for walking robots with four or two legs is still challenging due to the high complexity and redundancy of the underlying mechanical structure. Heuristic methods, e.g., inverted pendulum methods, usually do not consider the full dynamics of the robot as on-line computing of walking trajectories is computationally too expensive. Stating the problem of finding periodic and statically or dynamically stable gaits for legged robots off-

line avoids the problem of high on-line computational costs and allows for application of full three-dimensional dynamics models in the computation.
The optimal control problem is stated as follows [HvS03]:

$\min \mathcal{J}[\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t_f]$ s. t.	minimize the merit function $\mathcal{J}$ subject to
$\mathcal{M}\ddot{\mathbf{q}} = B\mathbf{u} - \mathcal{C} - \mathcal{G} + J_c^T \mathbf{f}_c$	system of MBS ODEs
$\mathbf{g_c}(\mathbf{q}) = 0$	contact algebraic conditions
$\mathbf{b}(\mathbf{q}_0, \mathbf{q}_f, t_0, t_f) = 0$	boundary conditions
$\mathbf{n}(\mathbf{q}, \mathbf{u}) \geq 0$	nonlinear inequality constraints,
$\mathbf{q}_{\min} \leq \mathbf{q} \leq \mathbf{q}_{\max},$	box constraints on state,
$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}$	and control variables.

The original DAE system (1) and (2) can be replaced for the numerical solution by the reduced dynamics equations, where only the states of the swing legs are included to the problems and the other states are obtained by the contact legs' inverse kinematics (see [Har99] for more details). Merit functions $\mathcal{J}$ may, e.g., be time t_f, energy or a weighted sum of both [HvS03]. Several nonlinear boundary and inequality conditions must be included to the problem. A detailed discussion of the constraints can be found in [BHK$^+$03] for a humanoid model and in [SHvS03] for a four-legged robot.

3.2 Optimization method

For solving the optimal control problem numerically, the method DIRCOL [vS01] is used. The states and controls are approximated by piecewise cubic resp. piecewise linear polynomials on a discrete and successively refinable time grid. The optimal control problem is thereby transcribed into a nonlinear program with the coefficients of the polynomials as variables, which may be solved by a – due to the special structure of the variables – sparse sequential quadratic programming method [GMS02]. For more details we refer to [vS01, HvS00].

4 Numerical and Experimental Results

This section shortly reviews results of the application of the presented approach for legged robots. Energy or time optimal walking trajectories involving full three-dimensional models have been calculated both for a MBS model of a four-legged robot [SHvS03] and a two legged humanoid robot [BHK$^+$03] (cf. Fig. 1). In [BHK$^+$03] and [SHvS03] originally SOAFOR, a set of Fortran subroutines implementing articulated body algorithm, spatial operator algebra and reduced dynamics for legged robots has been used, which has evolved from [Har99] but does not provide sensitivities. Here, we demonstrate that these solutions also can be obtained with almost the same efficiency using the much more flexible object-oriented modeling approach presented here.

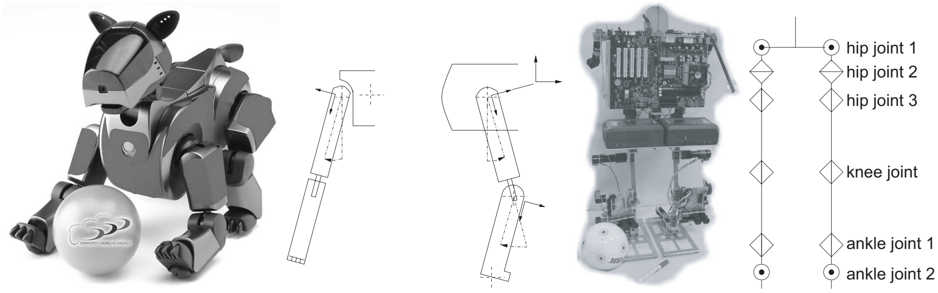

Figure 1: From left to right: Four legged Sony Aibo robot, kinematic structure of one leg of Aibo, prototype of a humanoid robot, kinematic structure of humanoid model. For both robots optimal walking trajectories have been calculated.

For Sony's four-legged robot model ERS-210A with legs of three degrees of freedom each, dynamics model data has been provided by the manufacturer. First optimal trajectories lead to successive refinements of the model: modified box constraints of applied torques and angular velocities of the joints lead to better fit the measured robot trajectories and slipping of the robot's feet on the ground was avoided by adding constraints on the horizontal contact forces [SHvS03]. Different symmetric gaits like trot and walk have been calculated.
The humanoid robot model, one torso and two 6 dof legs, of a real prototype developed in a collaboration including our group [BHK$^+$03] was applied for optimization of a two-phase half-stride consisting of a single limb support phase and a double limb support phase. Static stability explicitly is enforced by nonlinear inequality constraints. One dynamics evaluation for this system requires about 1.6 ms on a 2 GHz Pentium including exact sensitivities during smooth phases of the trajectory. As current humanoid control cycle times are around 1 ms, the approach enables the investigation of new nonlinear dynamics model-based control methods for humanoid robots by modular and flexible generation and re-using of code.

5 Conclusion

This paper has reviewed a new object-oriented architecture to provide multibody dynamics model computations which are indispensible for robot control applications. The main application considered for demonstration is trajectory optimization of legged robots. The approach consists of a carefully designed class hierarchy. This results in an efficient and light-weight code without a compile-to-code step required.

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