Modelling Artificial Consumer Markets - Cellular Automata vs. ODEs

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Abstract

This paper deals with a comparison between two modeling approaches for a Artificial Consumer Market (ACM). In a previous study the ACM has been implemented as an agent-based simulation environment using ordinary differential equations (ODEs). In this study cellular automata (CA) are used to model the core features of the ACM as the evolvement of the consumers' attitudes depending on the advertising budget of the firms. The cellular automata has been implemented using three different neighborhood functions: "Moore", "Hexagonal" and "Von Neumann". The results show that the cellular automata reproduces the properties of the ODEs for each of the neighborhood functions.

1 Introduction

This paper deals with a comparison between two modeling approaches for a Artificial Consumer Market (ACM). In a previous study the ACM has been implemented as an agent-based simulation environment using ordinary differential equations (ODEs). This simulation model already has been used for various optimization tasks for strategic marketing decisions and to explore numerous research hypothesis. The need for such models is based on the fact that in management science real world experiments are very rare and normally not feasible. Therefore the existence of well explored simulation models is essential and they are usefull for numerous applications.

In this paper cellular automata are used to model some stylized facts of the artificial consumer market mentioned above. The research question is how the differential equations of the ACM can be translated to proper rules for the cellular automata approach, to obtain the same properties as the model using ODEs. Cellular automata are based upon a discretization of space and time. Each cell holds a finite number of states and the temporal evolution of the automation is governed by transition rules which act locally and simultaneously on the cells. There are different neighborhood-functions which define the cells being determinant for updating the cell state. In this work the most common neighborhood-functions as 'Von Neumann', 'Moore' and 'Hexagonal' in 2-dimensional cellular automata are used and compared.

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2 Introduction to the ODE-based artificial consumer market

The artificial market is made up of a constant number of consumers. Each consumer has an individual aspiration point of attributes which the preferred product should possess. The choice process of the artificial consumers depends on the knowledge of the products offered at the market and the attitude the consumer gained by comparing his aspiration point to the perceived attributes of the product. The attitude depends to the advertising budget of a firm weighted by the price of the product. At initial time ($t_0 = 0$) no consumer agent knows anything about the products and the firms on the market. Primary through the advertising of the firms the consumers get information about the products and their attributes and so they are able to choose the best fitting product in a rational way. The success of each firm/product depends on the price, the attributes of the products and the invested advertising budget.

The following differential equation (ODE) shows the evolvement of the attitudes att_{ijk} of a consumer i regarding to the attribute k of product j:

$$\frac{d\,att_{ijk}(t)}{dt} = \frac{1}{price_j^*}\left[aif(budget_j)\,(1 - att_{ijk}(t))\; - b(t, budget_j)\,att_{ijk}(t)\right] \tag{1}$$

where $aif(budget_j)$ indicates the advertising impact function depending to the advertising budget of product j, $b(t, budget_j)$ characterize the forgetting rate of the consumer and $price_j^*$ refer the relative price of product j:

$$aif(budget_j) = e^{\alpha - \frac{\beta}{budget_j}} \qquad \text{and} \qquad price_j^* = \frac{price_j}{\frac{1}{J}\sum_{j=1}^{J}(price_j)}$$

In this equation the oblivion rate of advertised attributes is described by:

$$b(t, budget_j) = \frac{1}{1 + \mathcal{F}(t, budget_j)} \tag{2}$$

with

$$\Rightarrow \quad \mathcal{F}(t, budget_j) = budget_j(t) \cdot \int_{start}^{t} \frac{budget_j(\tau)}{\sum_j (budget_j(\tau))} \cdot e^{-b_0\,(t-\tau)} d\tau \tag{3}$$

Equation (2) describes the advertising effect of budget spent in the past. The amount of all advertising budgets effects the present consumers' attitude. The factor $b(t, budget_j)$ in the ODE (equ. 1) regulates the latency of advertising effects - in other words the oblivion of the consumer. So alternating advertising strategies are successfull because of the non-linear continuation of the effect former advertising budgets. If the advertising is stopped, after some time the effect vanishes and the oblivion rate of the consumers regarding the specific product increases and the attitude decreases continuously. In this paper this effect is modeled using the CA approach.

The utility of the consumer i with respect to each product j can be measured using the proportional distance between the appropriate aspiration point and the attitude corresponding to brand j and is calculated as

$$uti_{ij} = \frac{max(distance_{ij})}{distance_{ij}}. \quad (4)$$

This model has been implemented and further discussed in [5] and in a second publication at the ASIM 2005 by Jürgen Wöckl called: ”Agent-based Artificial Consumer Market and Optimization of Defensive Strategies”.

3 Design of the Cellular Automata

Using the CA approach to model the above mentioned artificial consumer market expressed through ODEs the space dimension of the cellular automata is not a metric distance between the cells in physical space. The ’space’ between cells and therefore also the neighborhood-function is defined in the attitude space of the consumer, which corresponds to the attribute space of the products. That means the K-dimensional attitude of the consumer moves in the K-dimensional attribute space of the products driven by the state variables price and budget of all firms in the market. The evolvement and the position in the attitude-space depends on the advertising budget spent and the price of the products set by the firms. To demonstrate the usableness and the equivalence of the cellular automata to the ODE - approach here primarily a 2-dimensional attitude/attribute - space is assumed to make the results graphically compareable.
The CA describes the evolvement of the consumers’ attitudes regarding a specific product. The cells of the cellular automata represents the amount and the distribution of the attitudes. As can been seen in equation (1 + 2), the evolvement depends mainly on the budget spent for advertising. The amount of budget spent in a period is connected to specific inflow cells of the CA. So a higher amount of budget effects to increase the overall amount of cell states representing a higher attitude. In this study the price and the claim of the firms has been fixed. If advertising is reduced or stopped the cell states of the CA will also be reduced over time concerning the update rules of the cellular automata.
The management decisions and strategies of the firms concerning price, claim and budget spent for advertising are subsumed as ”External Forces” influencing the evolvement of the attitudes modeled by the cellular automata (fig. 1). These external forces can be adopted over time but they are always influencing all consumers and therefore they can be seen as overall behavior of the external environment. In each period or generation of the CA the overall amount of the attitudes is evaluated and further is needed as an input amount for the adaption of the management decision for the next period.
The software of the original ACM using ODEs has been implemented in Matlab. The solution of the ODEs can be calculated using a simple Euler integrator as it is sufficient through the smooth of the adaption. The software used in this paper to implement the cellular automata is written in Python using some additional packages as numarray, SciPy and pylab. A general overview of the object-oriented implementation is presented in figure (2).

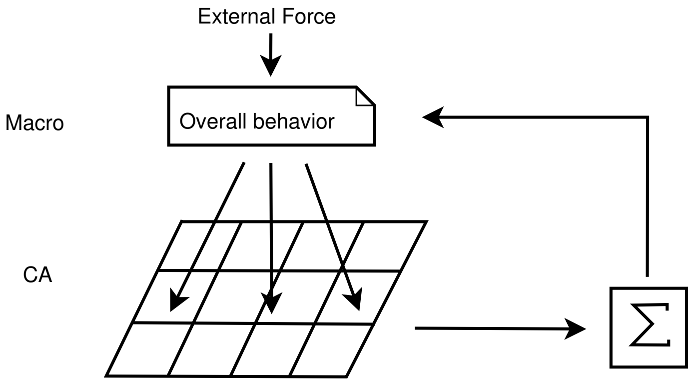

Figure 1: Design of the Cellular Automata coupled with external environment (see [4]).

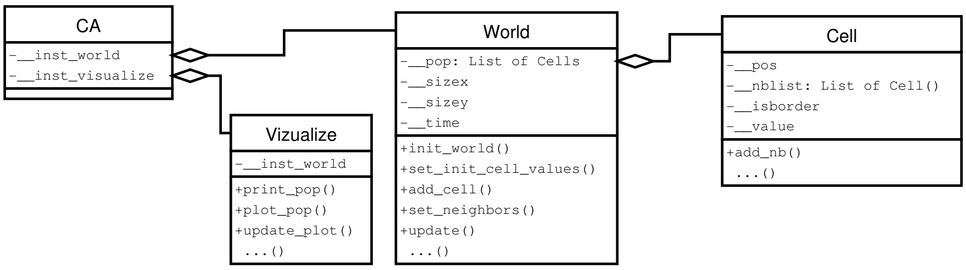

Figure 2: UML-Structure of the CA implementation. Due to a better clarity the set_.. and get_.. methods have been omitted.

4 Results

The CA is used to model the the learning and the oblivion about the products' features. The experiments with this simulation model show, that this approach can replace the original ODEs in the ACM. The initial conditions and a exemplary state of generation 80 of the CA is presented in figure (4). The CA is made up of a 10x10-grid which has been emphasized as sufficient to model the desired behavior of the CA. The inflow of the budget as a exogen set factor has been set to two randomly chosen cells. If the chosen cells are middle cells the evolvement is faster due to the higher number of neighbors and higher distance to the border with is kept at 0 to "cool" the system by providing an outflow. This enables the oblivion of the consumers if no budget is spent for advertising. This is required to model the properties of the ODE system. The selection of the inflowing cells is not critical and even border cell will work aside from a slower adaption rate.

Figure (3) shows the evolvement of the overall attitude and the dependence of the neighborhood function. It can be seen that the neighborhood function has some influence on the

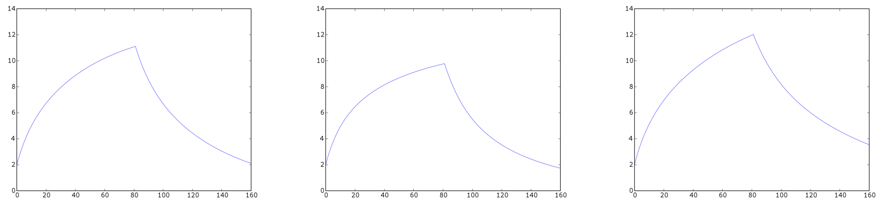

Figure 3: Evolvement of the overall attitudes of a individual consumer with different neighborhood functions; ”Moore” (left), ”Hexagonal” (middle), ”Von Neumann” (right). At ($t = 0$) a constant advertising budget is set and at ($t = 80$) the budget is totally phased out.

reached absolute level of the satisfaction, but the general form of the shape is similar.
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References

[1] Christian Buchta and Josef Mazanec. SIMSEG/ACM A Simulation Environment for Artificial Consumer Markets. Working Paper 79, SFB - Adaptive Modelling: www.wu-wien.ac.at/am, March 2001.

[2] Mark V. Lawson. *Finite automata*. CRC Press, 2004.

[3] Gary Lilien, Philip Kotler, and Sridhar Moorthy. *Marketing Models*. Prentice Hall International, 1992.

[4] Arthur Mynett and Qiuwen Chen. Cellular automata in ecological and ecohydraulics modelling. In *Cellular Automata*, 6th International Conference on Cellular Automata for Research and Industry, ACRI 2004, pages 502–512. Springer, 2004.

[5] Ulrike Schuster and Jürgen Wöckl. Optimal Defensive Strategies under Varying Consumer Distributional Patterns and Market Maturity. *Journal of Economics and Management*, 1(2):187 – 206, July 2005.

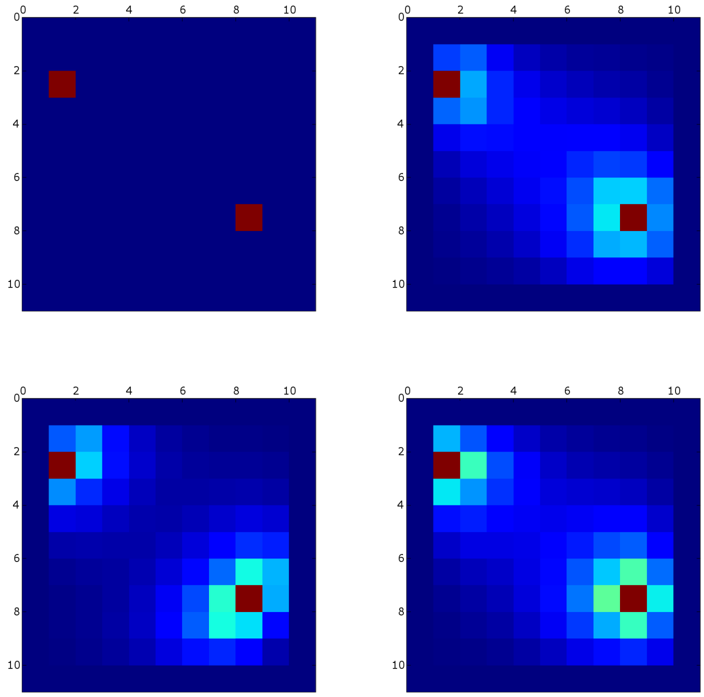

Figure 4: Snapshots of initial condition ($t = 0$) (top left) and population dynamics at ($t = 80$) with different neighborhood functions; ”Moore” (top right), ”Hexagonal” (down left), ”Von Neumann” (down right).

[6] Peter M.A. Sloot, Bastien Chopard, and Alfons G. Hoekstra, editors. *Cellular Automata*, 6th International Conference on Cellular Automata for Research and Industry, ACRI 2004. Springer, 2004.

[7] Dieter A. Wolf-Gladrow. *Lattice gas cellular automata and lattice Boltzmann models: an introduction*. Springer, 2001.