Viscoelastic Modeling of Human Tissue

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1 Introduction

The choice of the mathematical model to describe the viscoelastic behavior of human tissues is for numerical simulations of particular importance because of the obtainable accuracy. Especially within the medical profession exists a high interest in such simulations, since their aim is to understand certain effects within the human body more in detail and furthermore to be able to give forecasts as well as improvement opportunities for a medical intervention.
One of these medical problems represents the voice production after a larynx excision as consequence of e.g. laryngeal cancer. In this case the upper part of the esophagus, called the pharyngeal-esophageal (PE) segment, can be used as basis for a substitute voice. The geometry of this PE segment has hereby major influence on the engaging voice quality. During surgery the surgeon is able to form the geometry of this PE segment. But until now the surgeon has no precise guidelines or knowledge of how to shape it in an optimal way. The target of this project is to build a simulation tool which is capable to value different PE segment geometries in order to improve the quality of the substitute voice. A central point thereby is the choice of the mathematical model to describe the viscoelasticity of the vocal folds. The important phenomenon of viscoelasticity in the considered case is the damping effect under periodic excitation, as these has direct influence on the displacement amplitudes of the fold vibrations.

2 Mechanics

In order to give an overview of how mechanical damping can be considered appropriately the basic equations of mechanics as well as their numerical discretization with the finite element method (FEM) is discussed.

2.1 Basic Equations

The dynamical behavior of mechanical systems is described by Newton's law

$$\mathrm{DIV}\,[\boldsymbol{\sigma}] + \mathbf{f}_{\mathrm{V}} \quad = \quad \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}\,, \tag{1}$$

where $[\boldsymbol{\sigma}]$ denotes the Cauchy stress tensor, $\mathbf{f}_\mathrm{V}$ the mechanical volume force, ρ the mechanical density and $\mathbf{u}$ the mechanical displacement [1]. Using Voigt notation the first term in (1) can be expressed by the differential operator $\mathcal{B}$

$$\mathcal{B} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix}^T$$

so that DIV $[\boldsymbol{\sigma}] = \mathcal{B}^T\boldsymbol{\sigma}$. In the linear elastic undamped case the stress $\boldsymbol{\sigma}$ depends linearly on the strain $\boldsymbol{\varepsilon}$ and on the displacements $\mathbf{u}$ as follows $\boldsymbol{\sigma} = [\mathbf{D}]\boldsymbol{\varepsilon} = [\mathbf{D}]\mathcal{B}\mathbf{u}$. Here denotes $[\mathbf{D}]$ the elasticity tensor. The basic equations of mechanics are therefore

$$\mathcal{B}^T[\mathbf{D}]\mathcal{B}\mathbf{u} + \mathbf{f}_\mathrm{V} \quad = \quad \rho\frac{\partial^2\mathbf{u}}{\partial t^2}\,. \tag{2}$$

The relation between stress and strain is called the constitutive equation and represents the starting point of the considerations of damping effects. The models presented further on are different possibilities to take the damping through different constitutive equations into account.

2.2 Discretization of the Mechanical Equations with FEM

As the damping models are implemented within the FEM code CFS++ [2] the base of this method will be sketched in the following. For the reason of simplicity the boundary conditions of (2) are set to zero. Multiplying (2) by an appropriate test function $\mathbf{u}'$ and performing a partial integration will transform (2) to its variational formulation, which reads as follows: Find $\mathbf{u} \in \mathbf{H}_0^1$ such that

$$\int\limits_\Omega \rho\mathbf{u}' \cdot \ddot{\mathbf{u}}\ \mathrm{d}\Omega + \int\limits_\Omega (\mathcal{B}\mathbf{u}')^T[\mathbf{D}]\mathcal{B}\mathbf{u})\ \mathrm{d}\Omega = \int\limits_\Omega \mathbf{u}' \cdot \mathbf{f}_\mathrm{V}\ \mathrm{d}\Omega \tag{3}$$

for any $\mathbf{u}' \in \mathbf{H}_0^1$. This form can be decomposed into finite domains, called finite elements where for each element an element matrix can be created easily. Finally these element matrices are assembled to global system matrices. This procedure leads to the matrix form of (3) as

$$\mathbf{M}_u\underline{\ddot{u}} + \mathbf{K}_u\underline{u} = \underline{f}\,, \tag{4}$$

where $\mathbf{M}_u$ and $\mathbf{K}_u$ represent the assembled system matrices. The time discretization of that linear algebraic system of equations is performed by a standard Newmark scheme in an effective mass formulation [1].

3 Damping Models

Three damping models are discussed within this contribution. These models are the damping model according to Rayleigh, a rheological based model and a fractional damping model. Because the frequency range of vocal fold vibrations lies in the range of about 100 Hz to 1 kHz, this frequency scope is treated exclusively. The enquiries done so far as well as [3] attest the fractional damping model to be the most appropriate method for human tissue. Therefore the fractional damping is treated more in detail.

3.1 Rayleigh Damping

The idea behind the Rayleigh damping is a weighted addition of the global mass and the global stiffness matrix in order to obtain a global damping matrix [1]. The scalar weight of the stiffness matrix is α_R and that of the mass matrix is β_R.

$$\mathbf{C}_u = \alpha_R \mathbf{M}_u + \beta_R \mathbf{K}_u \tag{5}$$

This leads to the following matrix formulation, which is also solved with a Newmark scheme.

$$\mathbf{M}_u \underline{\ddot{u}} + \mathbf{C}_u \underline{\dot{u}} + \mathbf{K}_u \underline{u} = \underline{f}\,, \tag{6}$$

With the Rayleigh model the frequency behavior is however fixed to be $\xi = \frac{\alpha_R + \beta_R \omega^2}{\omega}$. Here ω denotes the angular frequency and ξ the modal damping parameter. For a high frequency the damping is thereby stiffness proportional und for a low frequency it is mass proportional. If one enforce a damping of $\xi = 10\%$ for 100 Hz and 1 kHz the resulting damping inbetween is shown in Fig. 1. This limits the applicability in time domain computations. For a frequency domain analysis different values of α_R and β_R for different frequencies can be realized because the frequencies are treated separately. Due to additional nonlinear effects such as contact of the PE segment one is restricted to a transient analysis.

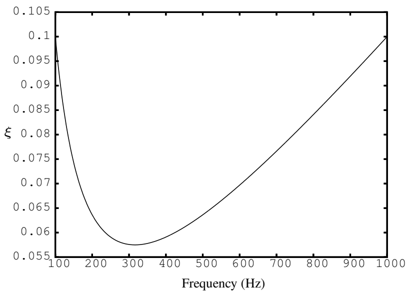

Figure 1: Damping ξ from 100 Hz to 1 kHz

3.2 Rheological Based Damping

The rheological based damping model as described by Krings [4] is here chosen to be implemented in CFS++. This model represents an enhancing of 1D rheological models like the Kelvin body into 3D. We also considered the possibility of different combinations for the deviatoric (shape changes) and for the hydrostatic (volume changes) part. With a Kelvin body for both parts the obtained damping matrix $\mathbf{C}_u$ is of the same shape as the stiffness matrix and a linear damping behavior over the frequency domain is obtained. This is the same behavior as the stiffness proportional part of the Rayleigh model and therefore also not adequate for vocal fold vibration as already discussed. In the future further combinations of damping models for the deviatoric and the hydrostatic part will be considered according to their frequency behavior.

3.3 Fractional Damping

The third damping model, which will be discussed, is the fractional damping model. This model is based on fractional derivatives which are therefore explained in the following.

3.3.1 Fractional Derivative

The fractional derivative according to Grünwald [3] is a generalization of the differentiation of integer order. Starting from a difference definition of the derivative of order 1, which reads as

$$\frac{d^1 f(t)}{dt^1} = \lim_{\Delta t \to 0} \frac{f(t) - f(t - \Delta t)}{\Delta t}, \tag{7}$$

$$\tag{8}$$

and by using the binomial coefficient we can write the derivative for any integer order n as

$$\frac{d^n f(t)}{dt^n} = \lim_{\Delta t \to 0} (\Delta t)^{-n} \sum_{j=0}^{n} (-1)^j \binom{n}{j} f(t - j\Delta t). \tag{9}$$

This form is restricted to integer orders n because of the binomial coefficient $\binom{n}{j}$. With the Gamma function $\Gamma(q)$, which is a generalization of the factorial, it is possible to obtain a formulation of this binomial coefficient, which is valid for any real number q. An abbreviated form can be given with the help of the Grünwald coefficients A_{j+1} [3].

$$(-1)^j \binom{q}{j} = \frac{\Gamma(j-q)}{\Gamma(-q)\Gamma(j+1)} = A_{j+1} \tag{10}$$

By inserting (10) into (9) we obtain an expression for the differentiation, which is valid for any real number q.

$$\frac{d^q f(t)}{dt^q} = \lim_{N \to \infty} (\frac{t}{N})^{-q} \sum_{j=0}^{N-1} A_{j+1} f(t - j\Delta t). \tag{11}$$

The Grünwald coefficients can be computed recursively in advance of the simulation to get an efficient implementation [5]

$$A_{j+1} = \frac{j-1-q}{j} A_j. \tag{12}$$

By considering the application of the fractional derivatives within fractional damping, we restrict q to positive values. According to (12) the series of the Grünwald coefficients A_{j+1} for $j \to \infty$ is strictly decreasing from the point where $j > q$, because the recursive multiplication factor is then smaller than 1. Furthermore is the limit of the monotonically decreasing Grünwald coefficients for $j \to \infty$ zero ($\lim_{j \to \infty} |A_{j+1}| = 0$). The fact that values are weighted less the further they lie in the past is known as *fading memory* and is consistent with the real material behavior. The *fading memory* effect motivates an approximative form of the derivative by cutting the series after N_l coefficients with $N_l < N-1$

$$\frac{d^q f(t)}{dt^q} \approx \lim_{N\to\infty} \left(\frac{t}{N}\right)^{-q} \sum_{j=0}^{N_l} A_{j+1} f(t - j\frac{t}{N})\,. \tag{13}$$

3.3.2 Fractional Damping

The concept of fractional derivatives can be used to generalize the rheological models, by replacing the integer order derivative by fractional order. Instead of springs and dashpots we consider a fractional element with the constitutive equation $\boldsymbol{\sigma} = \beta_F \frac{d^q}{dt^q}\boldsymbol{\varepsilon}$. Therefore the behavior of the fractional element is, despite the proportionality factor β_F, for $q = 1$ equivalent to a damper and for $q = 0$ equivalent to a spring. A common choice for the fractional derivative grade is a value between 0 and 1 [3]. Then the fractional element interpolates between the behavior of a spring and a dashpot. The fractional constitutive equation used in this contribution is a three parameter model

$$\boldsymbol{\sigma} + [\boldsymbol{\alpha_F}](\frac{d^q}{dt^q}\boldsymbol{\sigma}) = [\mathbf{D}]\boldsymbol{\varepsilon} + [\boldsymbol{\beta_F}](\frac{d^q}{dt^q}\boldsymbol{\varepsilon}\,). \tag{14}$$

The numerical discretization results in a form similar to (4). Different to the standard scheme (4) is however the stiffness matrix and the right hand side vector in which the history values are taken into account.
A 3D cube with edge length a is used in order to test the fractional damping model. The planes $x = 0, y = 0, z = 0$ of the cube are set to be fix and at the free corner (a, a, a) a force of amplitude $(f(t), f(t), f(t))$ with $f(t)$ like in Fig. 2 is acting so that a three dimensional stress condition is achieved. After $2/3$ of the total computation time the excitation force gets zero to obtain a free damped vibration.
First of all we will show results of a cube with $a = 10$ cm and a frequency of 100 Hz in order to clarify how many history values have to be considered, see Fig. 3. Compared

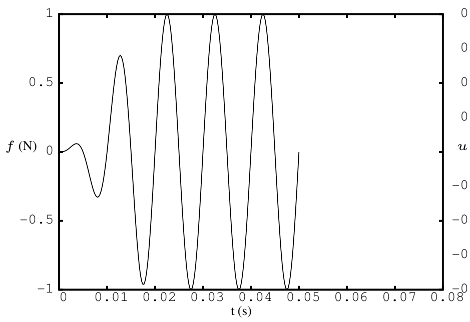

Figure 2: Excitation force

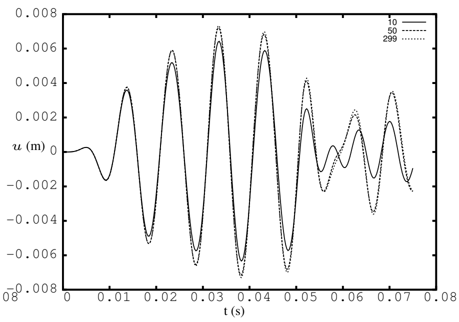

Figure 3: Varying the history values

to the case where all history values are stored (299) the error with 50 history values stays

small. This is valid for the forced vibration phase as much as for the free damped phase. So further computations were made with 50 history values.
As the fractional parameters of a specific material are in general not available they have to be identified. Therefore it is important to know what impact parameter variation has onto the damping behavior. For this reason parameter variations of α_F and β_F, as shown in Fig. 4, are made. In general the damping increases with decreasing α_F and increasing β_F.

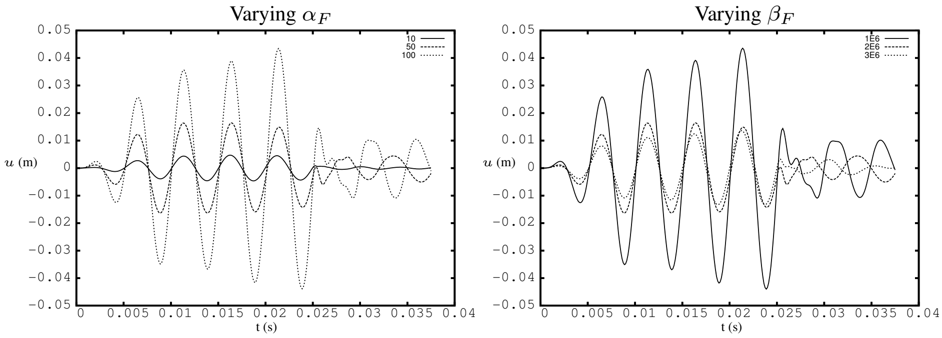

Figure 4: Parameter sensitivity of α_F (left) and β_F (right)

4 Conclusion

The most appropriate damping model to describe the viscoelasticity of human tissue is according to this investigation the fractional damping model. As for human tissue the parameter identification can not be done without measurements, the next step is to build a measurement setup. Therefore test items will be made to measure transfer functions.

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