A Parallel Algebraic Multigrid with Simultaneous Treatment of Multiple Right-Hand Sides for Lead Field Bases Computation in EEG and MEG Source Reconstruction.

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Abstract

Recent studies have shown the increased accuracy of realistic anisotropic high-resolution Finite Element (FE) method based volume conductor modeling in Electro- and Magneto-EncephaloGraphy (EEG/MEG) source reconstruction when comparing to spherical shell or Boundary Element (BE) approximations. If a software package were able to efficiently perform the necessary FE based forward computations within the inverse problem with even some millions of unknowns, then, besides the increased forward modeling accuracy, the appropriate FE meshing problem would be alleviated. In this paper, we will show how the application of a recently developed strategy, a parallel algebraic multigrid preconditioned conjugate gradient method with simultaneous treatment of multiple right-hand sides leads to an efficient and memory-economical computation of EEG and MEG lead field bases. The lead field bases are then used for an efficient treatment of the EEG/MEG inverse problem. We will show that for FE-meshes with some hundred thousand unknowns, the computations can now be performed in some few minutes on a single processor PC. The presented parallel approach furthermore offers the possibility to increase the resolution to even millions of unknowns so that in the future, the computations can even be performed on the voxel grid of the underlying Magnetic Resonance Imaging (MRI) modality.

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1 Introduction

The inverse problem in EEG/MEG aims at reconstructing the underlying current distribution in the human brain using potential differences and/or magnetic fluxes that are measured non-invasively directly, or at a close distance, from the head surface. The solution requires repeated computation of the forward problem, i.e., the simulation of EEG and MEG fields for a given dipolar source in the brain using a volume-conduction model of the head. The associated differential equations are derived from the Maxwell equations. Not only do various head tissues exhibit different conductivities, some of them are also anisotropic conductors as, e.g., skull, brain white and also gray matter. Until recently, there were no readily available methods that allow the determination of white matter conductivity anisotropy. Furthermore, there was a lack of sufficiently powerful software packages that would yield significant reduction of the computation time involved in such complex models hence satisfying the time-restrictions for the solution of the inverse problem.

In this paper, we used techniques of multimodal MRI in order to generate a high-resolution realistically shaped anisotropic volume conductor model. We improved the segmentation of the skull by means of a bimodal T1/PD-MRI approach. The eigenvectors of the conductivity tensors in anisotropic white matter were determined using whole head Diffusion-Tensor-MRI [Wol03].

When choosing the Finite Element (FE) method for volume conductor modeling, we need to solve a variety of large sparse systems of linear equations. Preconditioning techniques for the iterative solution process are then important to achieve the necessary accuracy and to speed the computation and keep the memory amount in reasonable limits. Multigrid based solution processes have proved to be of optimal order with respect to memory requirements and computational costs [Hac94]. It has recently been shown that especially Algebraic Multigrid based solver methods are appropriate for such kind of equations [HKR02, WKAR02, MV03]. If the resolution exceeds a limit of some hundred thousand nodes, parallelization gets an important concept. The most reasonable parallelization strategy in our case is the concept of distributing both memory and computational costs, because the institutions where EEG/MEG source localization is used as a diagnostic tool, are most often equipped with a cluster of PC's. For those purposes, we recently presented a parallel AMG preconditioned Conjugate Gradient (AMG-CG) solver for EEG and MEG source analysis [HKR02, WKAR02]. With regard to an efficient solution of the clinically relevant inverse problem, the construction of the so-called EEG and MEG lead field bases (fully populated matrices with "number of sensors many" rows and "number of finite element nodes" many columns) requires "number of EEG/MEG sensors" many solutions of FE equation systems in a setup phase for each individual head model [GA04, WGH04]. For this process, the "EEG/MEG sensors many" right-hand sides (block right-hand sides) can be precomputed and then simultaneously be treated by the solver. In this paper, we will discuss a recently developed strategy for a further speedup, the simultaneous treatment of multiple right-hand sides [HR05] and apply it to EEG/MEG source reconstruction. In [WARH04], we presented performance results for the computation of the lead field bases for a realistic anisotropic FE head model on a single processor machine. We compared the computation time for the construction of the EEG and MEG lead field bases for

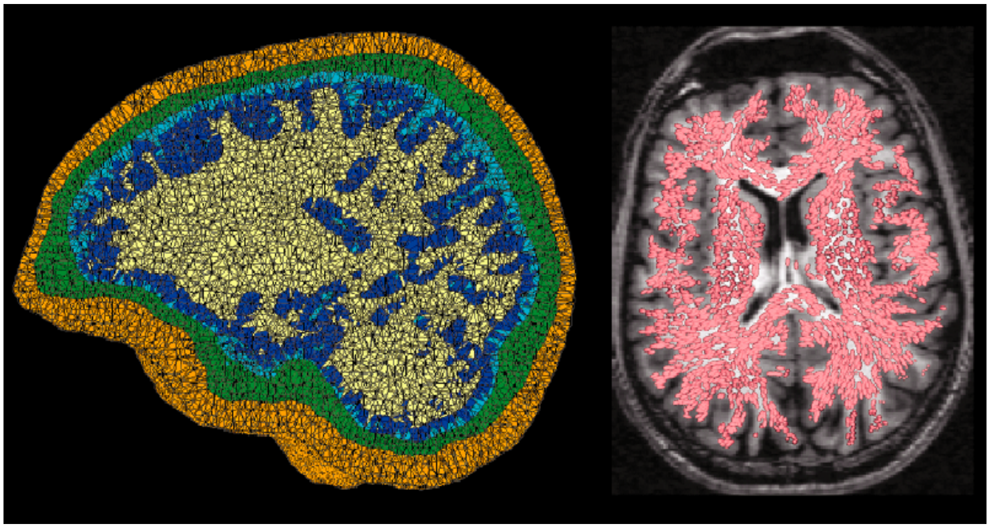

Figure 1: Left: Tetrahedral head model with 147,287 nodes and 892,115 elements. Right: Conductivity tensors in barycenters of white matter finite elements.

the Jakobi-preconditioned CG (J-CG), the symmetric Incomplete Cholesky preconditioned CG without fill-in (sym-IC(0)-CG) and the conventional AMG-CG [HKR02, WKAR02] with the serial version of the MultiRHS-AMG-CG while varying the number of simultaneously treated RHSs. In this paper, we will show the efficiency of the new block approach in a parallel computation on a PC cluster.

2 Methods

Compared to the AMG-CG presented in [HKR02, WKAR02], general algorithmical improvements were implemented for the new Multiple Right-Hand Side AMG-CG [HR05]. The old memory management for the stiffness and interpolation matrices was replaced by the classical Compact Sparse Row Storage (CRS) format in order to decrease the number of cache misses. Within the AMG algorithm, defect calculation follows forward Gauss-Seidel (GS) smoothing and both operations require matrix-vector operations. For symmetric stiffness matrices, parts of the matrix-vector operation from the last GS smoothing can be efficiently stored and reused by the defect calculation, a merging, which leads to a reduction of the operation count. The AMG procedure on the next coarser level is called with a zero-initial correction vector. This can be used, too, so that the first forward GS smoothing sweep on the coarser levels is reduced to half of the arithmetic and memory operations. If a V-cycle is chosen, with only one pre-smoothing sweep per level, then the special structure of this smoother on the coarser levels furthermore leads to a reduction of the subsequent defect computation.

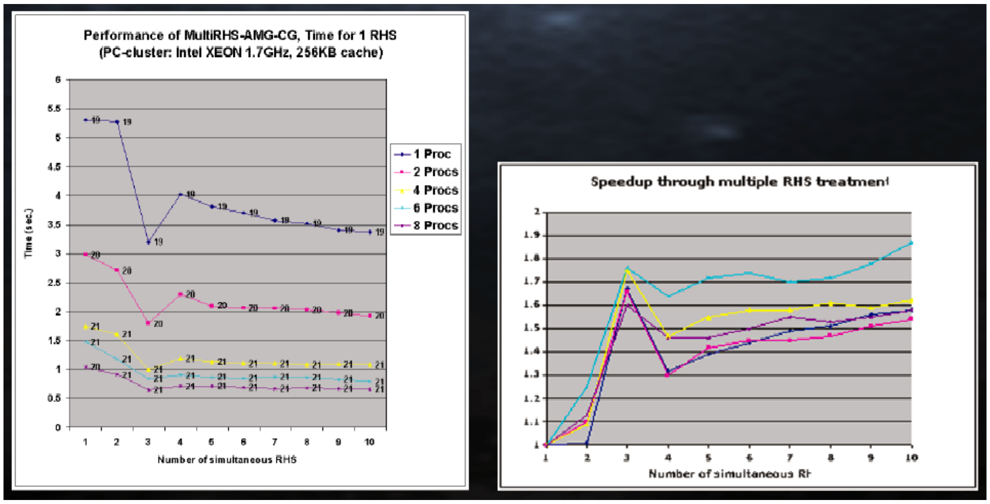

Figure 2: Time for the computation of one RHS on a Red-Hat Linux PC cluster using the new parallel MultiRHS-AMG-CG. The number of iterations is indicated above the curves. Right: Speedup through an improved cache hit rate using the new parallel MultiRHS-AMG-CG.

Since the RHSs in the lead field bases approach are computed beforehand and the stiffness matrix remains the same, we can simultaneously solve for a whole block of RHSs. The most computationally expensive operations in the AMG-CG are the matrix-vector operations within the CG and within the AMG components smoothing, defect calculation, interpolation and prolongation. If the vector for one RHS is exchanged against a whole block of vectors for multiple RHSs and if this block is not stored as a matrix, but as a long vector (first the first entry of all RHSs, then the second, etc.), then each matrix entry only has to be accessed once and can be multiplied to all corresponding values in the block-vector. This procedure results in much higher cache hit rates, which speeds the computations. For the simultaneous treatment of 3 RHS, the inner loops were manually unrolled, leading to a further reduction of the solver time.

3 Results

As a basis for our computations, we chose a realistic tetrahedral FE model with 147,287 nodes and 892,115 elements and anisotropic layers skull and white matter, a 71 electrode EEG and 147 channel MEG configuration (Fig.1).
The experiment was performed on a Red Hat Linux PC-cluster with 32 nodes, each equipped with a Xeon proc (1.7GHz, 256KB cache, 1GB memory) and a 1Gbit ethernet. We measured the performance of the parallel MultiRHS-AMG-CG forward simulations on

1, 2, 4, 6 and 8 nodes while varying the number of simultaneously treated RHSs (Fig.2, left). The number of iterations is shown over the curves (Fig.2, left). In Fig.2 (right), we evaluated the speedup of the parallel MultiRHS-AMG-CG through an improved cache hit rate. The simultaneous treatment of 3 RHSs leads to a speedup of at least 1.6 for all tested numbers of computational nodes.

4 Discussion

The combination of the lead field bases concept [GA04, WGH04] with the presented parallel MultiRHS-AMG-CG solver for the setup phase reduces significantly the complexity of high resolution anisotropic finite element head modeling. If FE meshes with some few hundred thousand nodes are used, the computation of the lead field bases in the setup phase for each individual head model can now be performed in a few minutes on a standard single processor PC. Furthermore, by means of the distribution of the memory on multiple computational nodes, our parallel approach allows the computation of the lead field bases for FE meshes with even millions of nodes in a reasonable time, so that the problem of FE meshing can be omitted by simply transforming each voxel from a segmented CT or MRI into a finite element. The FE forward approach then reduces to the multiplication of the lead field bases to the FE source load RHS vector [WGH04], which can easily be parallelized for FE computations with millions of unknowns. The presented concept can be used by all inverse methods in continuous or discrete source parameter space.
The treatment of multiple RHSs within the new MultiRHS-AMG-CG reduced the computation time for the EEG and MEG lead field by at least a factor of 2 (serial version, see [WARH04]) or 1.6 (parallel version). With the manual unrollment of inner loops for the simultaneous treatment of 3 RHSs, this approach belongs to the fastest on all platforms so that this choice of simultaneous RHSs is recommended in our software NeuroFEM-Pebbles ([Neu05, PEB02]). On single processor platforms with a smaller cache and a slower access to the main memory, the improvement of the data structures by means of the CSR storage for stiffness and interpolation matrices led to a further speedup factor of up to 1.38 (see [WARH04]). The source code of our software developments is available on a fee-free basis upon request.

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